ARCTIC APPLIED ALGEBRA

Schedule & Abstracts

Monday	Tuesday	Wednesday	Thursday	Friday
9:00 Registration	9:00 Marie-Françoise Roy	9:00 Antonio Lerario	9:00 Michel Coste	9:00 Christine Bachoc
10:00 Opening	10:00 Coffee break	10:00 Coffee break	10:00 Coffee break	10:00 Coffee break
10:30 Saugata Basu	10:30 Mohab Safey El Din	10:30 Nicolai Vorobjov	10:30 Tim Netzer	10:30 Frank Vallentin
11:30 Dmitry Novikov	11:30 Florian Kohl	11:30 Danielle Gondard	11:30 Tom Drescher	11:30 Maria Dostert
Lunch break	Lunch break		Lunch break	Lunch break
13:30 Greg Blekherman	13:30 Gunnar Fløystad		13:30 Gabriele Nebe	13:30 Kristian Ranestad
14:30 Victor Magron	14:30 A. Taveira Blomenhofer		14:30 Jan Rolfes	14:30 Coffee break
15:00 Coffee break	15:00 Coffee break	Excursion	15:00 Coffee break	15:00 Evelyne Hubert
15:30 Thorsten Theobald	15:30 Bernard Mourrain		15:30 Markus Kirschmer	16:00 Closing
16:30 Helen Naumann	16:30 Philipp Di Dio		16:30 Elif Segah Öztaş	
17:15 Reception & Poster session				
		20:00 Conference dinner		

Monday

9:00 - Registration.

10:00 - Opening.

10:30 - Saugata Basu. *Vandermonde varieties, mirror spaces, and cohomology of symmetric semi-algebraic sets.*
(Joint work with Cordian Riener)

The cohomology groups (with rational coefficients) of semi-algebraic sets defined by symmetric polynomials inherit a structure of a finite dimensional module over the symmetric group (from the action of the symmetric group on the ambient space). The isotypic decomposition of these modules shed important information on the Betti numbers of such sets, via the multiplicities of the various irreducible representations (the so called Specht modules), and the well known "hook formula" that gives the dimensions of these irreducible representations. We prove new vanishing results on the multiplicities of these Specht modules in the cohomology groups of semi-algebraic sets defined by symmetric polynomials (in each dimension). As a consequence we derive an algorithm for computing the first ℓ Betti numbers of such sets whose complexity is bounded polynomially in the number of variables and the number of defining polynomials, if the degrees of the defining polynomials, and ℓ are considered to be fixed. In contrast, the problem of computing even the zero-th Betti number of general semi-algebraic sets defined by polynomials of degrees bounded by some constant greater than one, is a PSPACE-hard problem (a result due to Reif). Our results are made possible by combining in a new way results from the cohomological study of mirror spaces due to Davis and Solomon, with the fundamental results on the topology of Vandermonde varieties due to Arnold, Giventhal and Kostov (as well as additional techniques).

11:30 - Dmitry Novikov. *Complex Cellular Structures.*
(Joint work with Gal Binyamini)

Real semialgebraic sets admit so-called cellular decomposition, i.e. representation as a union of cells homeomorphic to cubes, with very convenient homeomorphisms. Attempts to build a straightforward complex holomorphic generalization of this construction meet obstacles related to inner hyperbolic metric properties of holomorphic sets.

I will explain these obstacles and propose a complexification of the real cell decomposition for complex (sub)analytic and semialgebraic sets. Unlike resolution of singularities type results, the resulting cell decomposition depends uniformly on parameters and has polynomial complexity in semialgebraic case. As a corollary, we get polynomial in degrees and smoothness bounds in Yomdin-Gromov Algebraic Lemma.

Monday

13:30 - Greg Blekherman. *Symmetry and Nonnegativity.*

I will discuss several recent results on symmetric nonnegative polynomials and their approximations by sums of squares. I will consider several types of symmetry, but the situation is especially interesting in the limit as the number of variables tends to infinity. There are diverse applications to quantum entanglement, graph density inequalities and theoretical computer science.

14:30 - Victor Magron. *Exact polynomial optimization via SOS, SONC and SAGE decompositions.*

We consider the problem of certifying nonnegativity for certain classes of multivariate polynomials. We focus on finding exact sums of squares (SOS), sums of nonnegative circuits (SONC) and sums of arithmetic-geometric-mean-exponentials (SAGE) decompositions. For each case, we rely on existing relaxations: semidefinite programming (SDP) for SOS decompositions, geometric programming (GP) for SONC decompositions and relative entropy programming (REP) for SAGE decompositions.

For the SOS case, we provide a hybrid numeric-symbolic algorithm computing exact rational decompositions for polynomials lying in the interior of the SOS cone. It computes an approximate SOS decomposition for a perturbation of the input polynomial with an arbitrary-precision SDP solver. An exact SOS decomposition is obtained thanks to the perturbation terms.

We also provide two hybrid numeric-symbolic optimization algorithms, computing exact SONC and SAGE decompositions. Moreover, we provide a hybrid numeric-symbolic decision algorithm for polynomials lying in the interior of the SAGE cone. Each framework, inspired by previous contributions of Parrilo and Peyrl, is a rounding-projection procedure.

For a polynomial lying in the interior of the SOS or SAGE cone, we prove that the corresponding decision algorithm terminates within a number of arithmetic operations, which is polynomial in the degree and number of terms of the input, and singly exponential in the number of variables.

We also provide experimental comparisons between the implementation of these algorithms and available alternatives.

15:00 - Coffee break.

15:30 - Thorsten Theobald. *Irreducible infeasible subsystems of semidefinite systems.*

(Joint work with Kai Kellner and Marc Pfetsch)

Given real symmetric $n \times n$-matrices $A_0, \ldots, A_m$, let $A(y)$ denote the linear matrix pencil $A(y) = A_0 - \sum_{i=1}^{m} y_i A_i$. Farkas' lemma for semidefinite programming then characterizes feasibility of the system $A(y) \succeq 0$ in terms of an alternative spectrahedron. In the well-studied special case of linear programming, a theorem by Gleeson and Ryan states that the index sets of irreducible infeasible subsystems are exactly the vertices of the corresponding alternative polyhedron.

Here, we study the nonlinear setting of general spectrahedra. We show that one direction of this theorem can be generalized to the nonlinear situation of extreme points of general spectrahedra. The reverse direction, however, is not true in general, which we show based on studying the semialgebraic inequalities underlying the positive semidefiniteness condition of a certain counterexample. On the positive side, an irreducible infeasible block subsystem is obtained whenever the extreme point has minimal block support. Motivated by results from sparse recovery, we provide a criterion for the uniqueness of solutions of semidefinite block systems.

16:30 - Helen Naumann. *The dual cone of sums of non-negative circuit polynomials.*

(joint work with Mareike Dressler and Thorsten Theobald)

For a non-empty, finite subset $\mathcal{A} \subseteq \mathbb{N}_0^n$, denote by $C_{\mathrm{sonc}}(\mathcal{A}) \in \mathbb{R}[x_1, \ldots, x_n]$ the cone of sums of non-negative circuit polynomials with support $\mathcal{A}$. We derive a representation of the dual cone $(C_{\mathrm{sonc}}(\mathcal{A}))^*$ and deduce a resulting optimality criterion for the use of sums of non-negative circuit polynomials in polynomial optimization.

17:15 - Reception & Poster session.

- Martin Berger: *Analyzing Fibrin Networks Using Topological Data Analysis.*
- Laura Jakobsson: *Category of cellular resolutions: some properties and representation stability.*
- Milo Orlich: *Explicit Boij-Söderberg theory of ideals from a graph isomorphism reduction.*

Tuesday

9:00 - Marie-Françoise Roy. *Divide and conquer roadmaps for basic semi-algebraic sets.*
(Joint work with Saugata Basu)

In this new step of our long term project on the complexity of roadmaps, we were hoping to be able to use direclty our previous work on Divide and conquer roadmaps for algebraic sets *. Of course, part of it can be adapted but several new ideas and techniques were in fact needed. In particular, the best estimates are obtained for recursion trees which are not balanced.

10:00 - Coffee break.

10:30 - Mohab Safey El Din. *Faster roadmap algorithms for bounded real algebraic sets defined by quadrics.*
(Joint work with E. Schost)

A roadmap of a positive dimensional real algebraic set V is a real algebraic curve having a non-empty and connected intersection with all connected components of V. Hence, computing such a roadmap reduces the problem of counting the number of connected components of V to a similar problem in dimension 1. It can also be used to answer connectivity queries.

In this talk, assuming that $V \subset \mathbb{R}^n$ is smooth, bounded and given as the real solution set of a reduced regular sequence of p quadrics, we will present an algorithm computing a roadmap for V in time which is subexponential in n (but exponential in p).

11:30 - Florian Kohl. *Transfer-Matrix Methods meet Ehrhart Theory Abstract.*
(Joint work with Alexander Engström)

Counting (proper) colorings of graphs is a classical problem in combinatorics with connections to many different fields. In this talk, we want to examine proper k-colorings of graphs of the form $G \times P_n$, where G is any graph, and where P_n is the path graph on n nodes. There are two special cases, namely (1) where k is fixed but not n, and (2) where n is fixed but not k. In (1), the number of colorings can be determined using transfer-matrix methods, and in (2), the number of colorings can be counted using the chromatic polynomial/Ehrhart theory. We will use group actions and the symmetry of $G \times P_n$ to combine the two methods to get explicit formulas (depending on k and n) counting the number of colorings and we will give a restricted version of the famous reciprocity theorem for the chromatic polynomial. Furthermore, we will describe the doubly asymptotic behavior of graphs $G \times C_n$, where C_n is the cycle graph with n nodes as both n and k go to infinity.

Tuesday

13:30 - Gunnar Fløystad. *Monomial ideals from P-partitions.*

For a poset P a P-partition is an order-preserving map $P \to \mathbb{N}_0$. Denoting such maps as $\mathrm{Hom}(P, \mathbb{N}_0)$ this is itself naturally a poset. To any poset ideal (downset) $\mathcal{I}$ in $\mathrm{Hom}(P, \mathbb{N}_0)$ we associate two monomial ideals, the letterplace ideal $L(\mathcal{I}, P)$ and its Alexander dual, the co-letterplace ideal $L(P, \mathcal{I})$.

We inform on the omnipresence of such ideals and regular quotients of such ideals. They generalize strongly stable ideals, and initial ideals of determinantal ideals. The associated simplicial complex by the Stanley-Reisner correspondence gives balls whose boundaries comprehensively generalize Bier spheres. For quadratically generated letterplace ideals, they are smooth points on the Hilbert scheme and the full family of deformations can be described.

14:30 - Alexander Taveira Blomenhofer. *Identifying the support of measures from their moments using Sums-of-Squares Optimization.*

We study the problem of moment decomposition: Suppose the moments of a finitely supported measure are given up to a certain degree d. The task is to compute the support of the measure along with the weights of every point of nonzero measure. To solve this problem, it is necessary that the measure is uniquely determined by its moments of degree up to d. Particularly, in the case of $d = 3$ and at most dimension-many generic support vectors, it is possible to determine the support from an eigenvalue decomposition using only techniques from linear algebra. The underlying procedure, known as Jennrich's Algorithm, provides us in fact with an algorithmic proof that the measure is uniquely determined by its moments of order at most 3. However, there exists a substantial gap between the cases where an identification of the support could be possible in theory and the cases where current efficient algorithms such as Jennrich's are applicable. We develop a new algorithm which naturally extends Jennrich's algorithm to give approximation results for so-called overcomplete instances in which the support contains more than dimension-many elements.

15:00 - Coffee break.

15:30 - Bernard Mourrain. *Reconstruction of measures from moments.*

(Joint work with C. Josz and J.B. Lasserre)

We consider the problem of reconstructing a (finitely supported) measure from its moments in dimension n.

We describe first the algebraic Prony method for the decomposition of series of moments as sums of polynomial-exponentials. It exploits the algebraic structure of the Artinian Gorenstein algebra associated to the moment matrix and deduce the polynomial-exponential representation from eigenvectors of multiplication operators in the Artinian Gorenstein Algebra.

Decomposition problems reduces to discrete super-resolution problems. For this type of approach, exact recovery is guaranteed provided that a geometric spacing condition on the "supports" holds and the number of evaluations are sufficiently many (but not many). The semidefinite programming approach can then be applied.

We analyze and compare these approaches and illustrate their behavior on numerical experiments, related to sparse interpolation problems.

16:30 - Philipp Di Dio. *The truncated Moment Problem: Recent Advances in Carathéodory Numbers.*

The truncated moment problem became a key ingredient in several mathematical fields and applications such as optimization, sums of squares, non-negative polynomials, flat extension, cubature formulas, financial mathematics, data analysis, image classification, shape reconstruction/recognition, and signal processing to name only a few.

Since the fundamental result of Richter in 1957 that every truncated moment functional is represented by a finite sum of Dirac measures this minimal number, the Carathéodory number, attracted much attention. We give a short review of known results on the Carathéodory number and then go to the very recent advances. We present a large range of purely analytic to purely algebraic techniques to find and improve bounds on the Carathéodory number. Among other results we show that for every $\varepsilon > 0$ there is a moment functional $L : \mathbb{R}[x_1, \ldots, x_n]_{\leq d} \to \mathbb{R}$ which needs at least $(1 - \varepsilon) \cdot \binom{n+d}{n}$ point evaluations l_{x_i}. This solves an old open problem in flat extension theory: There are moment functionals on $\mathbb{R}[x_1, \ldots, x_n]_{\leq 2d}$ which must be extended to the worst degree $4d$ to have a flat extension.

We present a generalization of Carathéory numbers to mixtures of distributions such as Gaussian and Log-Normal distributions that is mostly used in financial mathematics, data analysis, and signal processing.

Wednesday

9:00 - Antonio Lerario. *Low degree approximation of real singularities.*

(Joint works with P. Breiding, D. N. Diatta and H. Keneshlou)

In this talk I will discuss some recent results that allow to approximate a real singularity given by polynomial equations of degree d (e.g. the zero set of a polynomial, or the number of its critical points of a given Morse index) with a singularity which is diffeomorphic to the original one, but it is given by polynomials of degree $O(d^{1/2}\log d)$. The approximation procedure is constructive (in the sense that one can read the approximating polynomial from a linear projection of the given one) and quantitative (in the sense that the approximating procedure will hold for a subset of the space of polynomials with measure increasing very quickly to full measure as the degree goes to infinity).

10:00 - Coffee break.

10:30 - Nicolai Vorobjov. *On irreducible components of real exponential hypersurfaces.*

(Joint work with Cordian Riener)

Fix any algebraic extension $\mathbb{K}$ of the field $\mathbb{Q}$ of rationals. In this article we study *exponential sets* $V \subset \mathbb{R}^n$. Such sets are described by the vanishing of polynomials with coefficients in $\mathbb{K}$, in n variables, and n exponential functions. Complements in $\mathbb{R}^n$ of exponential sets form a Noetherian topology in $\mathbb{R}^n$ which we will call Zariski topology. Let $P \in \mathbb{K}[X_1,\dots,X_n,U_1,\dots,U_n]$ be a polynomial such that $V = \{\mathbf{x} = (x_1,\dots,x_n) \in \mathbb{R}^n : P(\mathbf{x}, e^{x_1},\dots,e^{x_n}) = 0\}$. The main result of this paper states that, under Schanuel's conjecture over the reals, if the codimension of V is 1 and the real algebraic set Zer(P) is irreducible over $\mathbb{K}$, then either V is irreducible (with respect to Zariski topology) or every irreducible component of V of codimension 1 is a hyperplane through the origin over $\mathbb{Q}$. The family of all possible hyperplanes is determined by monomials of P. In case of a single exponential (i.e., when P is independent of $U_2,\dots,U_n$) the statements are stronger and independent of Schanuel's conjecture.

11:30 - Danielle Gondard. *On the number of connected components of smooth projective non empty real varieties.*

The talk present a formula giving the number of connected components of smooth projective non empty real varieties, in algebraic terms involving the function field of the variety. This is joint work with Eberhard Becker. A sketch of our original proof using R-places will give an opportunity to introduce related topics, as the real holomorphy ring and orderings of higher level, useful in the proof. We also give a characterization for R-places to be not in the same connected component of the space of $\mathbb{R}$-places. The references will mention the existence of two new proofs by Claus Scheiderer and J.-L. Colliot-Thélène (this last one is in german).

Wednesday

Excursion: Fjord-Tour by bus around the island Kvaløya.

12:15	Departure after the last talk from the campus (outside the conference building)
12:45	Arrival in Ersfjordbotn
	Possiblity to have lunch at Café Bryggejentene
14:30	Departure from Café Bryggejentene
15:15	Arrival in Sommarøy
	Free time to spend in the nature
	Possibility to have coffee at Sommarøy Arctic Hotel
17:15	Departure from Sommarøy
	Short Stop at Hella on the way back
18:45	Arrival in Tromsø

20:00 - Conference dinner. Restaurant Compagniet

Thursday

9:00 - Michel Coste. *Compactification of the group of rigid motions.*

(Joint work with Nestor Djintelbé)

The Study parametrization of the group of rigid motions gives a compactification of this group as the Study quadric in the projective space of dimension 7 with homogeneous equation $x_0y_0 + x_1y_1 + x_2y_2 + x_3y_3 = 0$. The boundary of this compactification is the 3-dimensional subspace $x_0 = x_1 = x_2 = x_3 = 0$, in which the information about the rotation part of rigid motions disappear. This information is recovered by blowing-up this subspace; one obtains in this way a 5-dimensional boundary in the product of projective spaces $\mathbb{P}^3 \times \mathbb{P}^3$.

This approach is used in order to compactify the workspace of parallel robots with restricted degrees of freedom. Among them, the three-legged 3-UPU robot with Tsai architecture which presents a rather mysterious operation mode (different from the purely translational and from the planar modes of operation). The boundary of this operation mode decomposes into irreducible components which are easy to analyse. This gives information on the kinematic behaviour of the robot in this operation mode, for sufficiently large lengths of legs.

10:00 - Coffee break.

10:30 - Tim Netzer. *Free Spectrahedra, Operator Systems and their Applications.*

Spectrahedra are the feasible sets of semidefinite optimization, and have recently gained a lot of attention in convex algebraic geometry. Many interesting facts can be learned by extending them to the non-commutative setup. These free extensions turn out to be special cases of operator systems, which provides a link to operator algebra. We will explain the concepts and some results in this interplay of areas. Interestingly, the results have applications, for example in optimization and theoretical quantum physics.

11:30 - Tom Drescher. *Operator Systems and Free Spectrahedral Coverings.*

Given a convex cone in the Euclidean space one can define two canonical operator systems associated to that cone: the maximal and the minimal system. It is known that only for few cones these systems can be represented as free spectrahedra. This raises the question how else the minimal and the maximal system relate to the free spectrahedra over the given cone. In the talk we will answer this question for the circular cone and give ideas on how this result might be extended to other three dimensional spectrahedral cones.

Thursday

13:30 - Gabriele Nebe. *Automorphisms of extremal lattices.*

The study of automorphisms of extremal lattices was motivated by similar research for binary self-dual codes. It started by considering extremal even unimodular lattices in dimensions which are multiples of 24. We know six such lattices, the Leech lattice in dimension 24, four lattices of dimension 48 and, since 2010, also one extremal lattice in dimension 72. One of the 48-dimensional lattices was found by a computer search for lattices with a certain automorphism of order 5. In his thesis Michael Jürgens extended the theory to automorphisms of modular lattices, aiming in the construction of an extremal 3-modular lattice in dimension 36. In the talk I will present new methods suitable for non unimodular lattices and results partly obtained in joint work with Markus Kirschmer.

14:30 - Jan Rolfes. *The covering number in different settings.*

The covering number is a fundamental concept in metric geometry. For a compact metric space (X, d) and a given radius r, it asks for the minimal number of r-balls needed to cover the space X.

In this talk, I focus on specific examples of metric spaces in the areas of geometry, probability theory and quantum computing. In particular, we discuss how recent approximation methods, inspired by moment methods in polynomial optimization, can be applied to these spaces.

15:00 - Coffee break.

15:30 - Markus Kirschmer. *Definite hermitian lattices with class number one.*

The local global-principle states that two hermitian spaces over some number field K are isometric if and only if they are isometric over every completion of K.

The genus of a lattice L in a hermitian space consists of those lattices which are isometric to L locally everywhere. Every genus decomposes into finitely many isometry classes. The number of isometry classes in a genus is called its class number. Hence the genera with class number one are precisely those lattices for which the local-global principle holds.

For indefinite lattices, the class number can be expressed a-priori in terms of some local invariants. For definite lattices, such a description is not possible. However, up to similarity, there are only finitely many genera with a given class number.

In the talk, I will present my classification of all definite hermitian lattices of class number one.

16:30 - Elif Segah Öztaş. *Some methods for generating DNA codes.*

In here, the methods which are lifted polynomials, coterm polynomials etc. to generate various kind of DNA codes are mentioned. Various algebraic structures are used as finite fields, chainrings, non-chain ring to generate DNA codes. There are reversible and reversible-complement strands in DNA.Thus the methods that are important, to obtain reversible and reversible-complement DNA codes and find optimal codes in algebraic structure. Moreover we obtain some optimal codes by real DNA genes.

Friday

9:00 - Christine Bachoc. *About a linear analogue of Freimans $3k-4$ theorem.*

(Joint work with Alain Couvreur and Gilles Zémor)

Freiman's $3k-4$ theorem states in precise terms that a subset of the integers with small doubling is necessary contained in a not too large arithmetic progression. We will discuss a multiplicative counterpart of this theorem in the context of a function field over an algebraically closed field, where the goal is to characterize the linear subspaces with small square. It turns out that in this situation, the Riemann-Roch spaces play the role of the arithmetic progressions. We will formulate a conjecture that can be seen as an analogue of Freiman's result and will sketch a proof of the case $k \leq 1$.

10:00 - Coffee break.

10:30 - Frank Vallentin. *Chromatic numbers of geometric graphs.*

A classical problem in discrete geometry (due to Hadwiger and Nelson) is to find the minimal number of colors one needs to color all points in the Euclidean plane so that no two points which are distance 1 apart receive the same color. Similar geometric coloring problems can be posed in the context of the hyperbolic plane, n-dimensional Euclidean spaces, $n-$dimensional spheres, or coloring the Voronoi tessellation of n-dimensional lattices. In this talk I will present a theoretical framework in which all these geometric coloring problems (assuming that the color classes are measurable sets) can be conveniently studied with the help of harmonic analysis and convex optimization.

11:30 - Maria Dostert. *Kissing number of the hemisphere in dimension 8.*

(Joint work with David de Laat and Philippe Moustrou)

The kissing number of spherical caps asks for the maximal number of pairwise non-overlapping unit spheres that can simultaneously touch a central spherical cap in n-dimensional Euclidean space. We consider especially the kissing number of the hemisphere in dimension 8. The kissing number of hemispheres provides less symmetries than the kissing number of unit spheres, which makes the problem more difficult. The kissing number problem of spheres coincides with the problem of finding a maximal spherical code with minimal angular distance $\frac{\pi}{3}$. The famous configuration of 240 points of unit spheres in dimension 8 given by the root lattice $E8$, which is an optimal spherical code of minimal angular distance pi/3, is unique up to isometry. From these 240 points we get a configuration on the hemisphere with 183 pairwise non-overlapping unit spheres. Bachoc and Vallentin determined an upper bound of 183.012 using semidefinite optimization, hence the kissing number of the hemisphere in dimension 8 is 183. Using the semidefinite program of Bachoc and Vallentin we obtain a sharp numerical bound of 183. In this talk, I will present how to get a rational solution of the semidefinite program by using the floating point solution of the SDP solver and the configuration given by the E8 lattice. Furthermore, I will provide a proof that this configuration of 183 points is unique up to isometry.

Friday

13:30 - Kristian Ranestad. *The adjoint of a polytope.*

(Joint work with Kathlen Kohn)

This talk concerns some algebraic geometric aspects of geometric modeling.

A definition of the adjoint of a polytope was given by Warren in 1996 in the context of geometric modeling. He defined this polynomial to generalize barycentric coordinates from simplices to arbitrary polytopes.

We prove a conjecture that the adjoint is the unique polynomial of minimal degree which vanishes on the non-faces of a simple polytope. In particular, we see that some polytopes can be deformed to singular non-minimal Calabi-Yau hypersurfaces on which the adjoint defines the unique canonical divisor.

14:30 - Coffee break.

15:00 - Evelyne Hubert. *Invariant algebraic sets and symmetrization of polynomial systems.*

Assuming the variety of a polynomial set is invariant under a group action, we construct a set of invariants that define the same variety. Our construction can be seen as a generalization of the previously known construction for finite groups once we introduce the symmetrizations of a polynomial w.r.t. a section to the orbits of the group action. The symmetrisations are polynomials in a generating set of rational invariants. The results have thus to be understood outside of a proper closed invariant variety, independent of the polynomial set considered.

16:00 - Closing.