Svar til oppgaver fra S.Log MATHEMA

Dette heftet inneholder svarforslag til oppgaver i Matematikk 2. Dette er ferskvare som oppdateres fortløpende, se datofeltet. Kalkulatorsvar er vist med CASIO fx-9860 emulator. Noen svar med MATLAB er også vist. Gi tilbakemelding om det oppdages feil eller andre rariteter.

2.6-1.abcdefg

a)

$1, 4, 9, 16, \ldots$ Tallfølge med kvadrattall

$1^2, 2^2, 3^2, 4^2, \ldots$ $a_n = n^2 \quad , \quad n = 1, 2, 3, \ldots$

b)

$3, 8, 13, 18, \ldots$ Differanse på 5 mellom hvert ledd

$5\cdot1-2, 5\cdot2-2, 5\cdot3-2, 5\cdot4-2, \ldots$ $a_n = 5n-2 \quad , \quad n = 1, 2, 3, \ldots$

$5\cdot0+3, 5\cdot1+3, 5\cdot2+3, 5\cdot3+3, \ldots$ $a_n = 5(n-1)+3 \quad , \quad n = 1, 2, 3, \ldots$

c)

$25, 21, 17, 13, \ldots$ Differanse på -4 mellom hvert ledd

$29-4\cdot1, 29-4\cdot2, 29-4\cdot3, 29-4\cdot4$ $a_n = 29-4\cdot n \quad , \quad n = 1, 2, 3, \ldots$ 25 – 4(n-1) ?

d)

$1/5, 1/25, 1/125, 1/625, \ldots$

$$a_n = \frac{1}{5^n} = 5^{-n} \quad , \quad n = 1, 2, 3, \ldots$$

e)

$1/3, 1/5, 1/7, 1/9, \ldots$

$$a_n = \frac{1}{2n+1} \quad , \quad n = 0, 1, 2, \ldots$$

f)

$$1, -\frac{1}{2}, \frac{1}{4}, -\frac{1}{8}, \ldots \quad >> \quad +\frac{1}{2^0} - \frac{1}{2^1} + \frac{1}{2^2} - \frac{1}{2^3}, \ldots \quad >> \quad \frac{(-1)^0}{2^0} + \frac{(-1)^1}{2^1} + \frac{(-1)^2}{2^2} + \frac{(-1)^3}{2^3}, \ldots$$

$$a_n = \left(-\frac{1}{2}\right)^{n-1} \quad , \quad n = 1, 2, 3, \ldots$$

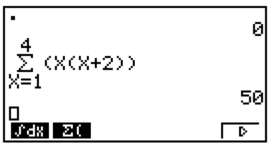
g)

$2, 0, 2, 0, \ldots$ >> $1+1, 1-1, 1+1, 1-1, \ldots$ $a_n = 1+(-1)^{n-1} \quad , \quad n = 1, 2, 3, \ldots$

2.6-2.abcdef

a)

$$\sum_{n=1}^{4} n(n+2) = 1(1+2)+2(2+2)+3(3+2)+4(4+2) = 3+8+15+24 = 50$$

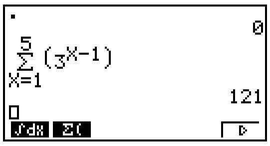



```
MATLAB/Octave numerisk
>> % Lager indekser fra 1 til 4
>> n = 1:4;
>> % Lager tallfølge av leddene i radvektor
>> an = n.*(n+2);
>> % Summen av leddene
>> sn = sum(an)
sn =   50
```

```
MATLAB symbolsk
>> syms m
>> symsum(m*(m+2), m, 1, 4)
ans =   50
```

b)

$$\sum_{k=1}^{5} 3^{k-1} = 3^0+3^1+3^2+3^3+3^4 = 1+3+9+27+81 = 121$$

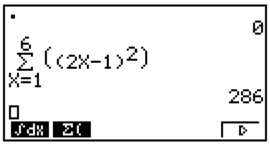



```
MATLAB/Octave numerisk
>> k = 1:5;             % indeksene
>> an = 3.^(k-1);       % rekkeleddene i en radvektor
>> sn = sum(an)         % summen av leddene i rekka
sn =   121

MATLAB symbolsk
>> syms k;
>> symsum(3^(k-1), k, 1, 5)
ans =   121
```

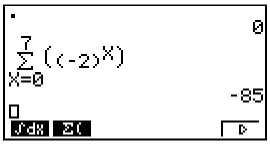
c)

$$\sum_{j=1}^{6} (2j-1)^2 = 1+9+25+49+81+121 = 286$$



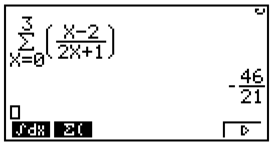
d)

$$\sum_{i=0}^{7} (-2)^i = 1-2+4-8+16-32+64-128 = -85$$



e)

$$\sum_{n=0}^{3} \frac{n-2}{2n+1} = \frac{0-2}{2\cdot 0+1}+\frac{1-2}{2\cdot 1+1}+\frac{2-2}{2\cdot 2+1}+\frac{3-2}{2\cdot 3+1} = \frac{-2}{1}+\frac{-1}{3}+0+\frac{1}{7} = -\frac{46}{21}$$

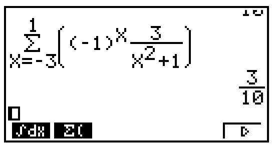


f)

$$\sum_{m=-3}^{1} (-1)^m \frac{3}{m^2+1}$$

$$=(-1)^{-3}\frac{3}{(-3)^2+1}+(-1)^{-2}\frac{3}{(-2)^2+1}+(-1)^{-1}\frac{3}{(-1)^2+1}+(-1)^{0}\frac{3}{0^2+1}+(-1)^{1}\frac{3}{1^2+1}$$

$$=-\frac{3}{10}+\frac{3}{5}-\frac{3}{2}+\frac{3}{1}-\frac{3}{2}=\frac{3}{10}$$




```
MATLAB/Octave numerisk
>> m = -3:1; format rat
>> am = (-1).^m .* 3./(m.^2+1);
>> sm = sum(am)
sm =   3/10
```

```
MATLAB symbolsk
>> syms m
>> symsum((-1)^m*3/(m^2+1), m, -3, 1)
ans =   3/10
```

2.6-3.abcdef

a)

$1+3+5+7=\sum_{n=1}^{4} 2n-1$ eller $1+3+5+7=\sum_{n=0}^{3} 2n+1$

b)

$$1+2+4+8+\ldots+128=2^0+2^1+2^2+2^3+\ldots+2^7=\sum_{n=0}^{7} 2^n$$

c)

$$3+7+11+15+19=3+(3+4)+(3+8)+(3+12)+(3+16)=\sum_{n=0}^{4} 3+4n=\sum_{n=1}^{5} 4n-1$$

d)

$$\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\frac{1}{9}+\frac{1}{11}+\frac{1}{13}=\sum_{n=1}^{6} \frac{1}{2n+1}$$

e)

$$1-\frac{1}{4}+\frac{1}{7}-\frac{1}{10}+\frac{1}{13}-\frac{1}{16}=\frac{1}{1}-\frac{1}{3+1}+\frac{1}{6+1}-\frac{1}{9+1}+\frac{1}{12+1}-\frac{1}{15+1}=\sum_{n=0}^{5} \frac{(-1)^n}{3n+1}$$

f)

$$2\cdot 5+3\cdot 7+4\cdot 9+\ldots+9\cdot 19=\sum_{n=2}^{9} n(2n+1)$$

2.6-4.ab

a)

Setter k = n-1, n = k + 1 og $\sum_{n=1}^{5}(n+2)=\sum_{k+1=1}^{k+1=5}(k+1+2)=\sum_{k=0}^{k=4}(k+3)$ **JA**

Eller ved beregning:

Venstre side, (1+2)+(2+2)+(3+2)+(4+2)+(5+2) = 3+4+5+6+7

Høyre side, (0+3)+(1+3)+(2+3)+(3+3)+(4+3) = 3+4+5+6+7 JA

b)

Setter k = n+2, n = k - 2 og $\sum_{n=-1}^{4} 2^{n-3}=\sum_{k-2=-1}^{k-2=4} 2^{k-2-3}=\sum_{k=1}^{k=6} 2^{k-5}$ **NEI**

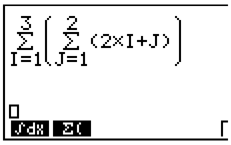
Eller, venstre side, $2^{-4}+2^{-3}+2^{-2}+2^{-1}+2^0+2^1$

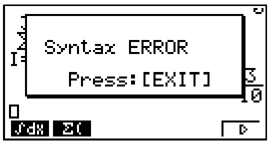
høyre side, $2^{-3}+2^{-2}+2^{-1}+2^0+2^1+2^2$ NEI

2.6-6.ab

a)

$$\sum_{i=1}^{3}\left(\sum_{j=1}^{2}(2i+j)\right)=2\cdot 1+1+2\cdot 1+2\ +\ 2\cdot 2+1+2\cdot 2+2\ +\ 2\cdot 3+1+2\cdot 3+2=33$$






```
MATLAB/Octave numerisk
>> sum = 0;
>> for ii = 1:3                    % Bruke ii og jj for ikke å
       for jj = 1:2                % omdefinere imaginær enhet
           sum = sum + 2*ii+jj;
       end
   end
>> sum
sum =  33
```

b)

$$\sum_{m=0}^{2}\left(\sum_{n=1}^{4} n\cdot 2^{m+n}\right)$$

$$=1\cdot 2^{0+1}+2\cdot 2^{0+2}+3\cdot 2^{0+3}+4\cdot 2^{0+4}$$
$$+1\cdot 2^{1+1}+2\cdot 2^{1+2}+3\cdot 2^{1+3}+4\cdot 2^{1+4}$$
$$+1\cdot 2^{2+1}+2\cdot 2^{2+2}+3\cdot 2^{2+3}+4\cdot 2^{2+4}$$
$$=686$$

```
MATLAB/Octave numerisk
>> for m = 0:2
       for n = 1:4
           sum = sum + n*2^(m+n);
       end
   end
>> sum
sum =  686
```

3.1-2.abcd

a) Nina står *ikke* til eksamen.

b) Det blåser *ikke* i dag. (Det er vindstille i dag)

c) $3+4\neq 7$

d) Trond har mindre enn 100 CD-er. $\overline{Antall\geq 100}$ <=> $Antall<100$

3.1-3.abcde

a) $'\,Kari\ har\ hvit\ kjole\,'\wedge'\,Kari\ har\ blå\ kjole\,'\wedge'\,Kari\ har\ rød\ kjole\,'$

b) $'\,Kari\ har\ mob\,'\wedge\neg('\,Kari\ har\ PC\,')$

Alt. $'\,Kari\ har\ mob\,'\wedge\overline{'\,Kari\ har\ PC\,'}$

c) $'\,Eksamen\ er\ grei\,'\vee'\,Rolf\ kjøper\ ny\ CD\,'$

d) $'\,Rolf\ bor\ hjemme\,'\vee'\,Rolf\ bor\ på\ hybel\,'$ (Men ikke samtidig?)

e) $\neg('Rolf\ er\ på\ skolen' \vee 'Rolf\ er\ hjemme')$

Alt. $\overline{'Rolf\ er\ på\ skolen' \vee 'Rolf\ er\ hjemme'}$

alt, $\neg('Rolf\ er\ på\ skolen') \wedge \neg('Rolf\ er\ hjemme')$

alt. $\overline{'Rolf\ er\ på\ skolen'} \wedge \overline{'Rolf\ er\ hjemme'}$

3.1-4.abcdef

a) $p \wedge q = S \wedge F = F$

b) $p \vee q = S \vee F = S$

c) $p \wedge \neg q = S \wedge (\neg F) = S \wedge S = S$

d) $\neg p \vee q = (\neg S) \vee F = F \vee F = F$

e) $\neg(\neg p \wedge q) = \neg(\neg S \wedge F) = \neg(F \wedge F) = \neg F = S$

f) $\neg(p \vee \neg q) = \neg(S \vee \neg F) = \neg S = F$

3.1-5.abcde

En implikasjon $p \rightarrow q$ kan formuleres muntlig på mange måter (hvis = dersom):

1) hvis *p*, så *q*	2) *p* er tilstrekkelig for *q*	3) *p* medfører *q*
4) *q* hvis *p*	5) hvis *p*, *q*	6) *q* hver gang når *p*
7) *p* bare hvis *q*	8) *q* er nødvendig for *p*	

a) $'Rosenborg\ vinner' \rightarrow 'de\ scorer\ flest\ mål'$ gir $q \rightarrow p$ - og IKKE $p \rightarrow q$

b) Tilsvarer 1), $p \rightarrow q$, JA

c) Tilsvarer 7), $p \rightarrow q$, JA

d) Ingen av de alternative formuleringene passer, omvendt implikasjon til 8)

e) Tilsvarer 1), $p \rightarrow q$, JA

3.1-6abc

a) $p \leftrightarrow q$

b) $(p \wedge q) \leftrightarrow r$

c) $(r \rightarrow q) \leftrightarrow p$

3.1-7.abceh

a)

p	q	$(p \vee q)$	$\neg q$	$(p \vee q) \wedge \neg q$
S	S	S	F	F
S	F	S	S	S
F	S	S	F	F
F	F	F	S	F

b)

p	q	$(p \wedge q)$	$\neg p$	$(p \wedge q) \rightarrow \neg p$
S	S	S	F	F
S	F	F	F	S
F	S	F	S	S
F	F	F	S	S

c)

p	q	$(p \vee q)$	$(p \wedge q)$	$(p \vee q) \rightarrow (p \vee q)$
S	S	S	F	S
S	F	S	F	F
F	S	S	F	F
F	F	F	S	S

e)

p	q	r	$(p \wedge q)$	$\neg r$	$(p \vee q) \vee \neg r$
S	S	S	S	F	S
S	S	F	S	S	S
S	F	S	F	F	F
S	F	F	F	S	S
F	S	S	F	F	F
F	S	F	F	S	S
F	F	S	F	F	F
F	F	F	F	S	S

h)

p	q	r	$(p \leftrightarrow q)$	$(p \vee r)$	$(p \leftrightarrow q) \rightarrow (p \vee r)$
S	S	S	S	S	F
S	S	F	S	S	F
S	F	S	S	S	F
S	F	F	S	S	F
F	S	S	S	S	F
F	S	F	F	F	F
F	F	S	S	S	F
F	F	F	S	F	S

3.1-8.a

Bruker sannhetstabeller

p	q	$(p \rightarrow q)$	$\neg(p \rightarrow q)$
S	S	S	F
S	F	F	S
F	S	S	F
F	F	S	F

p	q	$\neg q$	$p \wedge \neg q$
S	S	F	F
S	F	S	S
F	S	F	F
F	F	S	F

3.1-11.a

Bruker sannhetstabeller

p	q	$(p \rightarrow q)$	$(q \rightarrow p)$	$(p \rightarrow q) \vee (q \rightarrow p)$
S	S	S	S	S
S	F	F	S	S
F	S	S	F	S
F	F	S	S	S

3.1-12.abc

'regner' → 'stengt'
'regner'
.: 'stengt', Modus ponens

'skinnner' → 'bader'
¬'bader'
.: ¬'skinner', Modus tollens

'squash' → 'sliten'
'sliten' → 'vann'
.: 'squash' ' → 'vann' , Syllogisme

3.1-13.abcd

a) Ikke gyldig, feil bruk av mod.ponens?
b) Gyldig, modus tollens
c) Gyldig, først syllogismelov, så modus tollens
d) Gyldig, modus tollens

3.1-16.abcdef

a) Skriver n som oddetall, $n = 2m+1$
Omformer $n^2+2 = (2m+1)^2+2 = 4m^2+4m+1+2 = 2(2m2+2m+1) + 1$, et oddetall

b) Formulerer den kontrapositive påstanden,
'Hvis n er et liketall, så er $5n+1$ et oddetall.'
Skriver n som liketall, $n = 2m$
Omformer $5n+1 = 5\cdot2m+ 1 = 2(5m)+1$, et oddetall

c) Skriver m og n som liketall, $m=2u$, $n = 2v$
Omformer $3m+7n+1 = 3(2u)+7(2v)+1 = 6u+14v+1 = 2(3u+7v)+1$, et oddetall

d) Omformer $2mn +3 = 2(mn+1) +1$, et oddetall

e) Formulerer den kontrapositive påstanden,
'Hvis m og n begge er oddetall, så er mn et oddetall.'
Skriver m og n som oddetall, $m = 2u+1$ og $n=2v+1$
Omformer $mn = (2u+1)(2v+1) = 4uv+2u+2v+1 = 2(2uv+u+v)+1$, et oddetall.

f) Formulerer den kontrapositive påstanden,
'Hvis m og n er lik eller større enn 10, så er $m+n$ lik eller større enn 20.'
- og det er da vel riktig?

3.1-18.ab

a) Indirekte, kontrapositivt: 'Hvis n er et oddetall, så er $5n+2$ et oddetall.'
Setter $n=2k+1$, $5n+2=5(2k+1)+2=2(5k+3)+1$, et oddetall.

Selvmotsigelse, antar den motsatte konklusjonen,
'hvis $5n+2$ er et liketall, så er n et oddetall'.
Skriver n som generelt oddetall, $n=2k+1$, og setter inn for $5n+2$,
$5n+2 = 5(2k+1)+2 = 2(5k+3)+1$, et oddetall(!), men var forutsatt som liketall.

b) Indirekte, kontrapositivt: 'Hvis n er et oddetall, så er n^2+3 et liketall.'
Setter $n=2k+1$, $n^2+3=(2k+1)^2+3=2(2k^2+2k+2)$, et liketall.

Selvmotsigelse, antar den motsatte konklusjonen,
'hvis n^2+3 er et liketall, så er n et oddetall'.
Skriver n som generelt oddetall, $n=2k+1$, og setter inn for n^2+3 ,
$n^2+3 = (2k+1)^2+3=2(2k^2+2k+2)$, et liketall(!), men var forutsatt som oddetall.

3.1-20.ab

a) $n = -5$, $n^2 = 25$ (heltall kan være både negative og positive)

b) 1 + 3 = 4 (et liketall)

3.6-1.bdfh

b) Basistrinn, $n=1$ VS=4 HS=$1(3\cdot1+1) =4$ OK

Induksjonstrinn, *n*=*k* 4+10+16+...+(6*k*-2) = *k*(3*k*+1)

Induksjonstrinn, *n*=*k*+1 4+10+16+...+(6*k*-2) +(6(*k*+1)-2
= *k*(3*k*+1) + (6(*k*+1)-2
= 3k^2+7*k*+4 = (*k*+1)(3(*k*+1)+1)

d) Basistrinn, *n*=0 VS=1 HS= $\frac{3^{0+1}-1}{2}=1$ OK

Induksjonstrinn, *n*=*k* $3^0+3^1+3^2+\ldots+3^k=\frac{3^{k+1}-1}{2}$

Induksjonstrinn, *n*=*k*+1

$$3^0+3^1+3^2+\ldots+3^k+3^{k+1}$$
$$=\frac{3^{k+1}-1}{2}+3^{k+1}$$
$$=\frac{3\cdot 3^{k+1}-1}{2}=\frac{3^{(k+1)+1}-1}{2}$$

f) Basistrinn, *n*=1 VS=1·4=4 HS=1(1+1)^2=4 OK

Induksjonstrinn, *n*=*k* $1\cdot 4+2\cdot 7+3\cdot 10+\ldots+k(3k+1)=k(k+1)^2$

Induksjonstrinn, *n*=*k*+1

$$1\cdot 4+2\cdot 7+3\cdot 10+\ldots+k(3k+1)+(k+1)(3(k+1)-1)$$
$$=k(k+1)^2+(k+1)(3(k+1)+1)$$
$$=(k+1)((k+1)+1)^2$$

h) Basistrinn, *n*=1 VS=1/(1·5)=1/5 HS=1(4·1+1)=1/5 OK

Induksjonstrinn, *n*=*k*

$$\frac{1}{1\cdot 5}+\frac{1}{5\cdot 9}+\frac{1}{9\cdot 13}+\ldots+\frac{1}{(4k-3)(4k+1)}=\frac{k}{4k+1}$$

Induksjonstrinn, *n*=*k*+1

$$\frac{1}{1\cdot 5}+\frac{1}{5\cdot 9}+\frac{1}{9\cdot 13}+\ldots+\frac{1}{(4k-3)(4k+1)}+\frac{1}{(4(k+1)-3)(4(k+1)+1)}$$
$$=\frac{k}{4k+1}+\frac{1}{(4(k+1)-3)(4(k+1)+1)}$$
$$=\frac{k(4k+5)}{(4k+1)(4k+5)}+\frac{1}{(4k+1)(4k+5)}=\frac{4k^2+5k+1}{(4k+1)(4k+5)}$$
$$=\frac{(k+1)(4k+1)}{(4k+1)(4k+5)}=\frac{k+1}{4(k+1)+1}$$

3.6-2.bd

b) Basistrinn, *n*=2 VS=2^2=4 HS=2+1=3 OK

Induksjonstrinn, *n*=*k* $k^2>k+1$

Induksjonstrinn, *n*=*k*+1

$$(k+1)^2=k^2+2k+1>(k+1)+2k+1$$
$$(k+1)^2>(k+1)+1$$

d) Basistrinn, *n*=2 VS=2^2=4 HS=2!=2 OK

Induksjonstrinn, *n*=*k* $k^k>k!$

Induksjonstrinn, *n*=*k*+1

$$(k+1)^{(k+1)}=(k+1)(k+1)^k$$
$$(k+1)^{(k+1)}>(k+1)k^k$$
$$(k+1)^{(k+1)}>(k+1)\cdot k!$$
$$(k+1)^{(k+1)}>(k+1)!$$

3.6-3.ac

a) Basistrinn, *n*=1 VS=1^2=1 HS=1(1+1)(2+1)/6=1 OK

Induksjonstrinn, *n*=*m*

$$\sum_{k=1}^{m} k^2=\frac{m(m+1)(2m+1)}{6}$$

Induksjonstrinn, *n*=*m*+1

$$\sum_{k=1}^{m} k^2+(m+1)^2$$
$$=\frac{m(m+1)(2m+1)}{6}+\frac{6(m+1)^2}{6}$$
$$=\frac{2m^3+9m^2+13m+6}{6}$$
$$=\frac{(m+1)((m+1)+1)(2(m+1)+1)}{6}$$

c) Basistrinn, *n*=1 VS=1^3=1 HS=$1^2(1+1)^2$/4=1 OK

Induksjonstrinn, *n*=*m*

$$\sum_{k=1}^{m} k^3=\frac{m^2(m+1)^2}{4}$$

Induksjonstrinn, *n*=*m*+1

$$\sum_{k=1}^{m} k^2+(m+1)^3$$
$$=\frac{m^2(m+1)^2}{4}+\frac{4(m+1)^3}{4}$$
$$=\frac{m^4+6m^3+13m^2+12m+4}{4}$$
$$=\frac{(m+1)^2(m+1)^2}{4}$$

3.6-4.ace

a) Basistrinn, *n*=1 $3^{2\cdot 1}-1=9-1=8$ OK, delelig med 8

Induksjonstrinn, *n*=*k* $3^{2k}-1=8\cdot h$

Induksjonstrinn, *n*=*k*+1

$$3^{2(k+1)}-1=3^{2k+2}-1=9\cdot 3^{2k}-1$$
$$=9\cdot 3^{2k}-1=9\cdot 3^{2k}-9+8$$
$$=9(3^{2k}-1)-8=9\cdot 8h-8$$
$$=8(9h-1)$$

c) Basistrinn, *n*=1 $11^1-7^1=7$ OK, delelig med 7

Induksjonstrinn, *n*=*k* $11^k-4^k=7h$

Induksjonstrinn, *n*=*k*+1

$$11^{n+1}-4^{n+1}=11\cdot 11^{n}-4\cdot 4^{n}$$
$$=11\cdot 11^{n}-11\cdot 4^{n}+7\cdot 4^{n}$$
$$=11\left(11^{n}-4^{n}\right)+7\cdot 4^{n}$$
$$=11\cdot 7h+7\cdot 4^{n}$$
$$=7\left(11h+4^{n}\right)$$

e) Basistrinn, *n*=1 $\quad 1^{3}-4\cdot 1+6=3$ $\quad$ OK, delelig med 3

Induksjonstrinn, *n*=*k* $\quad k^{3}-4k+6=3h$

Induksjonstrinn, *n*=*k*+1

$$(k+1)^{3}-4(k+1)+6$$
$$=k^{3}+3k^{2}-k+3=\left(k^{3}-4k+6\right)+3k^{2}+3k-3$$
$$=3h+3\left(k^{2}+k+1\right)$$
$$=3\left(h+k^{2}+k+1\right)$$

3.7-6.ab

Viser ved induksjon

a) Basistrinn, *n*=2 $\quad a_1=1 \qquad a_1=2^{1}-1=1$

Induksjonstrinn, *n*=*k* $\quad a_k=2^{k}-1$

Induksjonstrinn, *n*=*k*+1

$$a_{k+1}=2\cdot a_k+1=2\left(2^{k}-1\right)+1$$
$$=2\cdot 2^{k}-1=2^{(k+1)}-1$$

b) Basistrinn, *n*=2 $\quad a_1=3 \qquad a_1=2^{1+1}-1=3$

Induksjonstrinn, *n*=*k* $\quad a_k=2^{k+1}-1$

Induksjonstrinn, *n*=*k*+1

$$a_{k+1}=2\cdot a_k+1=2\left(2^{k+1}-1\right)+1$$
$$=2\cdot 2^{k+1}-1=2^{(k+1)+1}-1$$

3.7-7.ac

a) Noen ledd,

$$a_1=0$$
$$a_2=a_1+3=3$$
$$a_3=a_2+3=6$$
$$a_4=a_3+3=9$$

Ser ut som om $\quad a_n=3(n-1)$, viser ved induksjon

Induksjonstrinn, *n*=*k* $\quad a_k=3(k-1)$

Induksjonstrinn, *n*=*k*+1

$$a_{k+1}=a_k+3$$
$$=3(k-1)+3$$
$$=3\left((k+1)-1\right)$$

c) Noen ledd,

$$a_1=0$$
$$a_2=a_1+2=2$$
$$a_3=a_2+3=2+3=5$$
$$a_4=a_3+4=2+3+4=9$$
$$a_5=a_4+5=2+3+4+5=14$$

Vi har fra tidligere,

$$1+2+3+...+n=\frac{n(n+1)}{2}$$

Trekker fra 1,

$$2+3+...+n=\frac{n(n+1)}{2}-\frac{2}{2}=\frac{(n-1)(n+2)}{2}$$

$$a_n=2+3+...+n=\frac{(n-1)(n+2)}{2}$$

3.7-8.bd

b) Noen ledd, $\{a_n\}=\{\ 1,3,5,7,...\}$

Oddetallsfølge, $a_n=a_{n-1}+2 \quad , \quad a_1=1$

d) Noen ledd, $\{a_n\}=\{\ 9,42,225,...\}$??

Ledd a_{n+1},

$$a_{n+1}=6^{n+1}+3(n+1)$$
$$=6\cdot6^n+3n+3$$
$$=6\cdot6^n+18n-15n+3$$
$$=6(6^n+3n)-15n+3$$
$$a_{n+1}=6a_n-15n+3 \quad , \quad a_1=9$$

6.2-1.ab

Når det har gått ett år øker forrige års kontobeløp med faktoren 1.05, men 3000 kr trekkes fra,

$$y_n=y_{n-1}\cdot1.05-3000 \quad , \quad n\geq1$$

Ordnet, $y_n-y_{n-1}\cdot1.05=-3000 \quad , \quad n\geq1$

Startverdien blir $y_0=30000$.

6.2-2.ab

Når det har gått ett år øker forrige års lønn med faktoren 1.03, og 8000 kr legges til,

$$y_n=y_{n-1}\cdot1.03+8000 \quad , \quad n\geq1$$

Ordnet, $y_n-y_{n-1}\cdot1.03=8000 \quad , \quad n\geq1$

Startverdien blir $y_0=300000$.

6.2-3

$y_n=2y_{n-1} \quad , \quad n\geq1$ >> $y_n-2y_{n-1}=0 \quad , \quad n\geq1$

6.2-4

$k=1$: $A_1=1$ [a]

$k=2$: $A_2=2\cdot 1$ [a,b] , [b,a]

$k=3$: $A_3=3\cdot 2\cdot 1$ [a,b,c], [a,c,b], [b,a,c], [b,c,a], [c,a], [a,c]

$k=n$: $A_n=n\cdot(n-1)\cdot\ldots\cdot 3\cdot 2\cdot 1=n\cdot A_{n-1}$

Likning, $A_n=n\cdot A_{n-1}$, ordnet $A_n-n\cdot A_{n-1}=0$, $n\geq 2$

Startverdi, $A_1=1$

6.2-5

k	*Mnd.antall*	*Mnd. sum*
$k=0$:	$A_0=0$	
$k=1$:	$A_1=1$	$S_1=1$
$k=2$:	$A_2=2$	$S_2=S_1+A_2=1+2=3$
$k=3$:	$A_3=3$	$S_3=S_2+A_3=3+3=6$
$k=n$:	$A_n=n$	$S_n=S_{n-1}+n$

Likning, $S_n=S_{n-1}+n$, ordnet $S_n-S_{n-1}=n$, $n\geq 1$

Startverdi, $S_0=0$

6.2-6

Se også eksempel 6.5 i læreboka.

Med *like antall* *b*-er forstås 0 stk *b*-er, 2 stk *b*-er, 4 stk *b*-er osv.

For $k=1$: [**a**] [b] [**c**] Like antall (0) *b*-er, $y_1=2$

For $k=2$: [**aa**] [ab] [**ac**] [ba] [bb] [bc] [**ca**] [**cb**] [**cc**] 4+1=5 like antall (0, 2) *b*-er, $y_2=5$

For $k=3$: [**aaa**] [aab] [**aac**] [aba] [**abb**] [abc] [**aca**] [acb] [**acc**] 8 stk 0 like
[baa] [**bab**] [bac] [**bba**] [bbb] [**bbc**] [bca] [**bcb**] [bcc] + 6 stk 2 like
[**caa**] [cab] [**cac**] [cba] [**cbb]** [cbc] [**cca**] [ccb] [**ccc**] $y_3=8+6=14$

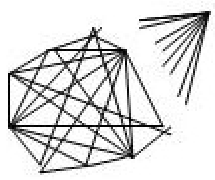
For $k=4$ Vi får OK ord ved at *a* eller *c* legges til ordene ovenfor, det blir $2\cdot y_3$ ord. Dessuten vil en *b* lagt til de ugyldige ordene ovenfor også gi et OK ord, det skulle bli 3^3-y_3 ord, i alt $y_4=2y_3+3^3-y_3$.

For $n=k$: $y_n=y_{n-1}+3^{n-1}$

Likning, $y_n-y_{n-1}=3^{n-1}$, $n\geq 2$

Startverdi, $y_1=2$

6.2-7



Det kan trekkes y_{n-1} linjer mellom $n-1$ punkter. Legger vi til et punkt slik at vi har en oppstilling med n punkter, kan det trekkes $y_{n-1}+n-1$ fra det nye punktet til hvert av de andre punktene.

$y_n=y_{n-1}+n-1$, $n\geq 4$ >> $y_n-y_{n-1}=n-1$, $n\geq 4$

Startverdi er $y_3=3$.

6.2-8

Oppgaven er som 6.2-6 , med utvidelse av antall bokstaver til 8, $y_n=(8-1)y_{n-1}+8^{n-1}-y_{n-1}$

Likning, $y_n-6y_{n-1}+8^{n-1}$, $n\geq 1$

Startverdi, $y_0=1$ (Uklart om et kodeord kan være tomt, *n*=0?)

6.3-1.abcdef

a) Orden 1, ikke homogen pga. ikke-null på høyre side

b) Orden 2, **ok**, homogen, lineær med k.koeffisient 1, -3

c) Orden 1, homogen, lineær, ikke konstant koeffisient pga .koeffisient *n*

d) Orden 2, homogen, ikke-lineær pga. y_{n-1}^2

e) Orden 4, **ok**, homogen, lineær med k.koeffisient 1, -5, 2, 1

f) Orden 7, **ok**, homogen, lineær med k.koeffisient 1, 4, 0, 0, 0, 0, 6

6.3-2.abcdef

a) Karakteristisk likning, $\lambda-4=0$, $\lambda=4$, $y_n=A\cdot 4^n$

b) Karakteristisk likning, $\lambda+2=0$, $\lambda=-2$, $y_n=A\cdot(-2)^n$

c) Karakteristisk likning, $\lambda^2-4=0$, $\lambda_1=-2$, $\lambda_2=2$, $y_n=A\cdot 2^n+B\cdot(-2)^n$

d) Karakteristisk likning, $\lambda^2-5\lambda+6=(\lambda-2)(\lambda-3)=0$,

$\lambda_1=2$, $\lambda_2=3$, $y_n=A\cdot 2^n+B\cdot 3^n$

e) Karakteristisk likning, $\lambda^2+\lambda-2=(\lambda-1)(\lambda+2)=0$, $\lambda_1=1$, $\lambda_2=-2$,

$y_n=A\cdot 1^n+B\cdot(-2)^n=A+B\cdot(-2)^n$

f) Karakteristisk likning, $4(\lambda^2+2\lambda+\frac{3}{4})=4((\lambda+\frac{1}{2})(\lambda+\frac{3}{2}))=0$, $\lambda_1=-\frac{1}{2}$, $\lambda_2=-\frac{3}{2}$

$y_n=A\cdot(-\frac{1}{2})^n+B\cdot(-\frac{3}{2})^n$

6.3-3.abcdef

a) Karakteristisk likning , $\lambda^2-6\lambda+9=(\lambda-3)(\lambda-3)=0$ $\lambda_1=3$, $\lambda_2=3$,

$y_n=A\cdot 3^n+B\cdot n\cdot 3^n$

b) Karakteristisk likning , $\lambda^2-2\lambda+1=(\lambda-1)(\lambda-1)=0$ $\lambda_1=1$, $\lambda_2=1$,

$y_n=A\cdot 1^n+B\cdot n\cdot 1^n=A+B\cdot n$

c) Karakteristisk likning , $4(\lambda^2+\lambda+\frac{1}{4})=4(\lambda+\frac{1}{2})(\lambda+\frac{1}{2})=0$ $\lambda_1=-\frac{1}{2}$, $\lambda_2=-\frac{1}{2}$

$y_n=A\cdot(-\frac{1}{2})^n+B\cdot n\cdot(-\frac{1}{2})^n$

d) Karakteristisk likning , $\lambda^2+4=(\lambda-2i)(\lambda+2i)=0$ $\lambda_1=2i$, $\lambda_2=-2i=2e^{-\frac{\pi}{2}i}$,

$y_n=r^n[A\cos(n\phi)+B\sin(n\phi)]=2^n[A\cos(n\frac{\pi}{2})+B\sin(n\frac{\pi}{2})]$

e) Karakteristisk likning , $\lambda^2+2\lambda+4=(\lambda+1+\sqrt{3}i)(\lambda+1-\sqrt{3}i)=0$

$\lambda_1=1+\sqrt{3}i$, $\lambda_2=1-\sqrt{3}i$, $r=2$, $\phi=\frac{\pi}{3}$

$y_n=r^n[A\cos(n\phi)+B\sin(n\phi)]=2^n[A\cos(n\frac{\pi}{3})+B\sin(n\frac{\pi}{3})]$

f) Karakteristisk likning, $\lambda^3-\lambda^2-4\lambda+4=(\lambda-2)(\lambda+2)(\lambda-1)=0$

$\lambda_1=2$, $\lambda_2=-2$, $\lambda_3=1$

$y_n=A\cdot 2^n+B\cdot(-2)^n+C$

6.3-4.abcd

a) Karakteristisk likning, $\lambda-6=0$, $\lambda=6$, generelt $y_n=A\cdot 6^n$

Initialverdi, $y_0=3=A\cdot 6^0=A$

Løsning, $y_n=3\cdot 6^n$

b) Karakteristisk likning, $\lambda^2-2\lambda-3=(\lambda+1)(\lambda-3)=0$, $\lambda_1=-1$, $\lambda_2=3$

Generell løsning, $y_n=A\cdot(-1)^n+B\cdot 3^n$

Initialverdi $n=0$, $y_0=3=A\cdot 1+B\cdot 1=A+B$

Initialverdi $n=1$, $y_1=5=A\cdot(-1)+B\cdot 3=-A+3B$ >> $A=1$, $B=2$

Løsning, $y_n=(-1)^n+2\cdot 3^n$

c) Karakteristisk likning, $\lambda^2+4\lambda+4=(\lambda+2)(\lambda+2)=0$, $\lambda_1=-2$, $\lambda_2=-2$

Generell løsning, $y_n=A\cdot(-2)^n+B\cdot n\cdot(-2)^n$

Initialverdi $n=0$, $y_0=1=A\cdot 1+B\cdot 0\cdot 1=A$

Initialverdi $n=1$, $y_1=2=A\cdot(-2)+B\cdot 1\cdot(-2)=-2A-2B$ >> $A=1$, $B=-2$

Løsning, $y_n=(-2)^n-2n(-2)^n=(-2)^n(1-2n)=evt=(-2)^n+n(-2)^{n+1}$

d) Karakteristisk likning, $\lambda^2-2\lambda+2=(\lambda-1+i)(\lambda-1-i)=0$,

$\lambda_1=1-i$, $\lambda_2=1+i$ $r=\sqrt{2}$, $\phi=\frac{\pi}{4}$

Generell løsning, $y_n=r^n[A\cos(n\phi)+B\sin(n\phi)]=(\sqrt{2})^n[A\cos(n\frac{\pi}{4})+B\sin(n\frac{\pi}{4})]$

Initialverdi $n=0$, $y_0=1=1(A\cos 0+B\sin 0)=\sqrt{2}A$ >> $A=1$

Initialverdi $n=1$, $y_1=1=\sqrt{2}(A\cos\frac{\pi}{4}+B\sin\frac{\pi}{4})=\sqrt{2}(1\cdot\frac{1}{\sqrt{2}}+B\cdot\frac{1}{\sqrt{2}})$ >> $B=0$

Løsning, $y_n=(\sqrt{2})^n(\cos(n\frac{\pi}{4})+0)=2^{\frac{n}{2}}\cos(n\frac{\pi}{4})$

6.3-5.abcd

Ser først på løsning av det homogene problemet, det kan få innvirkning på y_p , $y_n^{(h)}=A\cdot 2^n$

a) $y_n^{(p)}=K\cdot n+L$

b) $y_n^{(p)}=K\cdot n^3+L\cdot n^2+M\cdot n+N$

c) $y_n^{(p)}=K\cdot 3^n+L$

d) $y_n^{(p)}=K\cdot n\cdot 2^n$ (Likhet med homogen løsning)

6.3-6.abc

a) $y_n+2y_{n-1}=3n^2-n$, $n\geq 1$, $y_0=1$

Rot i karakteristisk likning, $\lambda=-2$ $y_n^{(h)}=A(-2)^n$

Partikulær løsning, $y_n^{(p)}=Kn^2+Ln+M$

Setter $y_n^{(p)}$ inn i likninga, $Kn^2+Ln+M+2(K(n-1)^2+L(n-1)+M)=3n^2-n+0$

$3Kn^2+(3L-4K)n+2K-2L+3M=3n^2-n+0$

$K=1$, $L=1$, $M=0$

Foreløpig løsning, $y_n=y_n^{(h)}+y_n^{(p)}=A(-2)^n+n^2+n$

Startverdi, $y_0=1=A(-2)^0+0+0$ >> $A=1$

Løsning, $y_n=(-2)^n+n^2+n$

b) $y_n-3y_{n-1}=2n+3-2^n$, $n\geq 1$, $y_0=1$

Rot i karakteristisk likning, $\lambda=3$ $y_n^{(h)}=A\cdot 3^n$

Partikulær løsning, $y_n^{(p)}=Kn+L+M\cdot 2^n$

Setter $y_n^{(p)}$ inn i likninga, $Kn+L+M\cdot 2^n-3(K(n-1)+L+M\cdot 2^{(n-1)})=2n+3-2^n$

$-2Kn+3K-2L+M\cdot 2^n-3M\cdot 2^{n-1}=2n+3-2^n$

$-2Kn+3K-2L-\frac{1}{2}M\cdot 2^n=2n+3-2^n$

$K=-1$, $L=-3$, $M=2$

Foreløpig løsning, $y_n=y_n^{(h)}+y_n^{(p)}=-n-3+2\cdot 2^n+A\cdot 3^n$

Startverdi, $y_0=1=-0-3+2\cdot 2^0+A\cdot 3^0$ >> $A=2$

Løsning, $y_n=-n-3+2^{n+1}+2\cdot 3^n$

c) $y_n-y_{n-1}=6n$, $n\geq 1$, $y_0=2$

Rot i karakteristisk likning, $\lambda=1$ $y_n^{(h)}=A\cdot 1^n=A$

Partikulær løsning, $y_n^{(p)}=K\,n+\,L$

Partikulær løsning og homogen løsning har samme ledd, justerer derfor

$y_n^{(p)}=K\,n^2+\,L\,n$

Setter $y_n^{(p)}$ inn i likninga, $K\,n^2+\,L\,n-(K(n-1)^2+\,L(n-1))=6\,n+\,0$

$K\,n^2+\,L\,n-K\,n^2+2\,K\,n-K-L\,n+\,L=6\,n+\,0$

$K=3$, $L=3$

Foreløpig løsning, $y_n=y_n^{(h)}+\,y_n^{(p)}=A+\,3\,n^2+\,3\,n$

Startverdi, $y_0=2=A+\,0+\,0$ >> $A=2$

Løsning, $y_n=3\,n^2+\,3\,n+\,2$

6.3-7.abcd

Ser først på løsning av det homogene problemet, det kan få innvirkning på y_p ,
karakteristisk likning, $\lambda^2-2\,\lambda-3=(\lambda+\,1)(\lambda-3)=0$, $y_h=A\cdot(-1)^n+\,B\cdot 3^n$

a) $y_n^{(p)}=K\cdot n^2+\,L\cdot n+\,M$

b) $y_n^{(p)}=K\cdot 4^n+\,L\cdot 2^n$, generell konstant i stedet for 1

c) $y_n^{(p)}=K\cdot n\cdot 3^n+\,L$, likhet med homogen løsning, øker grad av *n*

d) $y_n^{(p)}=(K\cdot n^2+\,L\,n)(-1)^n$, likhet med homogen løsning, øker grad av *n*

6.3-8.abcd

a) $y_n-5\,y_{n-1}+\,6\,y_{n-2}=5\cdot 4^n$

Karakteristisk likning, $\lambda^2-5\,\lambda+\,6=(\lambda-2)(\lambda-3)=0$, $\lambda_1=2$, $\lambda_2=3$

Homogen løsning, $y_n^{(h)}=A\cdot 2^n+\,B\cdot 3^n$

Partikulær tips, $y_n^{(p)}=K\cdot 4^n$

Setter $y^{(p)}$ inn i likn. $K\,4^n-5\,K\,4^{n-1}+\,6\,K\,4^{n-2}=5\cdot 4^n$

$K-5\,K\,4^{-1}+\,6\,K\,4^{-2}=5$ >> $K=40=10\cdot 4^1$

Løsning, $y_n=A\cdot 2^n+\,B\cdot 3^n+\,10\cdot 4^1\cdot 4^n=A\cdot 2^n+\,B\cdot 3^n+\,10\cdot 4^{n+1}$

b)

$y_n+\,5\,y_{n-1}+\,6\,y_{n-2}=12\,n-2(-1)^n$

Karakteristisk likning, $\lambda^2+\,5\,\lambda+\,6=(\lambda+\,2)(\lambda+\,3)=0$, $\lambda_1=-2$, $\lambda_2=-3$

Homogen løsning, $y_n^{(h)}=A\cdot(-2)^n+\,B\cdot(-3)^n$

Partikulær tips, $y_n^{(p)}=K\,n+\,L+\,M\,(-1)^n$

Setter $y^{(p)}$ inn i likn.

$$K\,n+\,L+\,M\,(-1)^n+\,5(K\,(n-1)+\,L+\,M\,(-1)^{n-1})+\,6(K\,(n-2)+\,L+\,M\,(-1)^{n-2})=12\,n-2(-1)^n$$

$$12\,K\,n-17\,K+\,12\,L+\,2\,M\,(-1)^n=12\,n-2\,(-1)^n \quad >> \quad K=1 \;,\; L=\frac{17}{12} \;,\; M=-1$$

Løsning, $y_n = A\cdot(-2)^n + B\cdot(-3)^n + n - (-1)^n + \frac{17}{12} = A\cdot(-2)^n + B\cdot(-3)^n + n + (-1)^{n+1} + \frac{17}{12}$

c)

$$y_n - 6\,y_{n-1} + 8\,y_{n-2} = 3\cdot 5^n - 2^n$$

Karakteristisk likning, $\lambda^2 - 6\lambda + 8 = (\lambda-2)(\lambda-4) = 0$, $\lambda_1 = 2$, $\lambda_2 = 4$

Homogen løsning, $y_n^{(h)} = A\cdot 2^n + B\cdot 4^n$

Partikulær tips, $y_n^{(p)} = K\cdot 5^n + L\cdot n\cdot 2^n$, likhet med hom. løsning, øker grad av *n*

Setter $y^{(p)}$ inn i likn.

$$K\cdot 5^n + L\cdot n\cdot 2^n - 6\left(K\cdot 5^{n-1} + L\cdot(n-1)\cdot 2^{n-1}\right) + 8\left(K\cdot 5^{n-2} + L\cdot(n-2)\cdot 2^{n-2}\right) = 3\cdot 5^n - 2^n$$

$$-L\cdot 2^n + \frac{3}{25}K\cdot 5^n = 3\cdot 5^n - 2^n \quad >> \quad L = 1 \;, \quad K = 25 = 5^2$$

Løsning, $y_n = A\cdot 2^n + B\cdot 4^n + 5^{n+2} + n\cdot 2^n = B\cdot 4^n + 5^{n+2} + (A+n)2^n$

d)

$$y_n - 2\,y_{n-1} + y_{n-2} = 6\,n$$

Karakteristisk likning, $\lambda^2 - 2\lambda + 1 = (\lambda-1)(\lambda-1) = 0$, $\lambda_1 = 1$, $\lambda_2 = 1$

Homogen løsning, $y_n^{(h)} = A\cdot 1^n + B\cdot n\cdot 1^n = A + B\,n$ (Sammenfallende røtter)

Partikulær tips, $y_n^{(p)} = K\cdot n^2$, likhet med homogen løsning, øker grad av *n*

Setter $y^{(p)}$ inn i likn. $K\cdot n^2 - 2\left(K\cdot(n-1)^2\right) + K\cdot(n-2)^2 = 6\,n$

Dette gir ikke løsning, øker grad av *n* en gang til,

Partikulær tips 2, $y_n^{(p)} = K\cdot n^3 + L\cdot n^2$

Setter $y^{(p)}$ inn i likn. $K\cdot n^3 + L\cdot n^2 - 2\left(K\cdot(n-1)^3 + L\cdot(n-1)^2\right) + K\cdot(n-2)^3 + L\cdot(n-2)^2 = 6\,n$

$$6\,K\,n - 6\,K + 2\,L = 6\,n \quad >> \quad K = 1 \;, \quad L = 3$$

Løsning, $y_n = n^3 + 3\,n^2 + B\,n + A$

6.3-10.ab

a) Setter y_n til fondbeløp etter n år, $y_n = 1.05\cdot y_{n-1} - 1$

Rekursjonslikning, $y_n - 1.05\,y_{n-1} = -1$, $n \geq 1$, $y_0 = 25$

b) Rot i karakteristisk likning, $\lambda = 1.05$, $y_n^{(h)} = A\cdot 1.05^n$

Partikulær løsning, $y_n^{(p)} = K$

Setter $y_n^{(p)}$ inn i likninga, $K - 1.05\,K = -1$ >> $K = \frac{-1}{1-1.05} = 20$

Foreløpig løsning, $y_n = y_n^{(h)} + y_n^{(p)} = 20 + A\cdot 1.05^n$

Startverdi, $y_0 = 25 = 20 + A\cdot 1.05^0 = 20 + A$ >> $A = 5$

Løsning, $y_n = 5\cdot 1.05^n + 20$

Setter tiden det tar for å oppnå 35 millioner til x ,

$y_x = 35 = 5 \cdot 1.05^x + 20$ og løser som ikke-diskret likning,

$1.05^x = \frac{35-20}{5} = 3$ >> $x = \frac{\ln 3}{\ln 1.05} = 22.5$

Løsning: Det tar 23 år og fondet er da $5 \cdot 1.05^{23} + 20 = 35.4$ mill

6.3-11.ab

a) Setter y_n til lånebeløp etter n år, $y_n = 1.04 \cdot y_{n-1} - 50$ i KNOK

Rekursjonslikning, $y_n - 1.04\, y_{n-1} = -50$, $n \ge 1$, $y_0 = 500$

b) Rot i karakteristisk likning, $\lambda = 1.04$, $y_n^{(h)} = A \cdot 1.04^n$

Partikulær løsning, $y_n^{(p)} = K$

Setter $y_n^{(p)}$ inn i likninga, $K - 1.04\,K = -50$ >> $K = \frac{-50}{1-1.04} = 1250$

Foreløpig løsning, $y_n = y_n^{(h)} + y_n^{(p)} = 1250 + A \cdot 1.04^n$

Startverdi, $y_0 = 500 = 1250 + A \cdot 1.04^0 = 1250 + A$ >> $A = -750$

Løsning, $y_n = 1250 - 750 \cdot 1.04^n$

Setter tiden det tar for å oppnå 0 KNOK til x ,

$y_x = 0 = 1250 - 750 \cdot 1.04^x$ og løser som ikke-diskret likning,

$1.04^x = \frac{1250}{750} = \frac{5}{3}$ >> $x = \frac{\ln 5 - \ln 3}{\ln 1.04} = 13.02$

Løsning: Det tar ~13 år.

6.3-12.ab

a) Setter y_n til antall etter n timer, $y_n = 3 \cdot y_{n-1}$

Rekursjonslikning, $y_n - 3\, y_{n-1} = 0$, $n \ge 1$, $y_0 = 120$

b) Rot i karakteristisk likning, $\lambda = 3$

Foreløpig løsning, $y_n = A \cdot 3^n$

Startverdi, $y_0 = 120 = A \cdot 3^0 = A$ >> $A = 120$

Løsning, $y_n = 120 \cdot 3^n$

7.4-1.abcdef

a)

$$\lim_{x \to 2} \frac{x^3 - 8}{x - 2} = \left(\frac{0}{0}\right) = \lim_{x \to 2} \frac{3x^2}{1} = \underline{\underline{12}}$$

b)

$$\lim_{x \to 1} \frac{\cos(\pi x) + 1}{x - 1} = \left(\frac{-1+1}{1-1}\right) = \left(\frac{0}{0}\right) = \lim_{x \to 1} \frac{-\pi \sin(\pi x)}{1} = \frac{0}{1} = \underline{\underline{0}}$$

c)

$$\lim_{x \to 0} \frac{e^{3x} - e^{-3x}}{\sin(2x)} = \left(\frac{1-1}{0}\right) = \left(\frac{0}{0}\right) = \lim_{x \to 0} \frac{3e^{3x} + 3e^{-3x}}{2\cos(2x)} = \frac{6}{2} = \underline{\underline{3}}$$

d)

$$\lim_{x\to 0}\frac{\sin^{-1}x-2x}{x-\tan^{-1}x}=\left(\frac{0-0}{0-0}\right)=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{(1-x^2)^{-\frac{1}{2}}-2}{1-(1+(2x)^2)^{-1}\cdot 2}=\frac{1-2}{1-2}=\underline{\underline{1}}$$

e)

$$\lim_{x\to 0}\frac{e^{4x}-1}{5x^2}=\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{4e^{4x}}{10x}=\left(\frac{4}{0}\right)\quad \underline{\underline{\text{Grenseverdi eksisterer ikke}}}$$

f)

$$\lim_{t\to 0}\frac{e^t-\cos t}{1-\sqrt{t+1}}=\frac{'0'}{'0'}=\lim_{t\to 0}\frac{e^t+\sin t}{-\frac{1}{2}(t+1)^{-\frac{1}{2}}\cdot 1}=\frac{1}{-\frac{1}{2}}\quad \underline{\underline{-2}}$$

7.4-5.abcde

a)

$$\lim_{x\to\infty}\frac{5x^4+4x^2+6}{1-2x-x^4}=\lim_{x\to 0}\frac{5+\frac{4}{x^2}+\frac{6}{x^4}}{\frac{1}{x^4}-\frac{2}{x^3}-1}=\frac{5+0+0}{0-0-1}=\ \underline{\underline{-5}}$$

alternativt

$$\lim_{x\to\infty}\frac{5x^4+4x^2+6}{1-2x-x^4}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to 0}\frac{20x^3+8x}{-2-4x^3}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to 0}\frac{60x^2+8}{-12x^2}=\left(\frac{\infty}{\infty}\right)$$

$$=\lim_{x\to 0}\frac{120x}{-24x}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to 0}\frac{120}{-24}=\ \underline{\underline{-5}}$$

b)

$$\lim_{x\to\infty}\frac{-2x^2+2x-1}{7x^2-4x^3}=\lim_{x\to 0}\frac{\frac{-2}{x}+\frac{2}{x^2}-\frac{1}{x^3}}{\frac{7}{x}-4}=\frac{-0+0-0}{0-4}=\frac{0}{-4}=\underline{\underline{0}}$$

c)

$$\lim_{x\to\infty}\frac{-8x^6-x+3}{2x^5+7x^3-2}=\lim_{x\to 0}\frac{-8x-\frac{1}{x^4}+\frac{3}{x^5}}{2+\frac{7}{x^2}-\frac{2}{x^5}}$$

$$=\lim_{x\to 0}\frac{-8x}{2+\frac{7}{x^2}-\frac{2}{x^5}}+\lim_{x\to 0}\frac{\frac{-1}{x^4}+\frac{3}{x^5}}{2+\frac{7}{x^2}-\frac{2}{x^5}}='\infty'+'0'\text{ , Grenseverdi eksisterer ikke}$$

d)

$$\lim_{z\to\infty}\frac{5\ln z+4z}{3\ln z-7z}=\left(\frac{\infty+\infty}{\infty-\infty}\right)=\lim_{z\to\infty}\frac{5\frac{1}{|z|}+4}{3\frac{1}{|z|}-7}=-\underline{\underline{\frac{4}{7}}}$$

e)

$$\lim_{x\to\frac{\pi}{2}^-}\frac{\ln(\cos x)}{\tan x}=\left(\frac{-\infty}{\infty}\right)=\lim_{x\to\frac{\pi}{2}^-}\frac{\frac{1}{\cos x}\cdot(-\sin x)}{\frac{1}{\cos^2 x}}=\lim_{x\to\frac{\pi}{2}^-}-\sin x\cdot\cos x=\underline{\underline{0}}$$

7.10-1.def

d)

Argumentet til logaritmefunksjonen må være >0. Argumentet er null for $y=4\mathrm{x}^2$ som kan framstilles som en parabelkurve. De punktene (*x*,*y*) som ligger over denne kurven har $y>x^2$ som gir argument >0 og som tilhører definisjonsmengden.

e)

Invers cosinus må ha argument i området [-1 , 1]. Siden x^2+y^2 er null eller positiv, betyr det at $x^2+y^2=1$, som kan framstilles av grafen til en sirkel med radius 1 er det geometriske stedet for punkter (*x*,*y*) som akkurat oppfyller $x^2+y^2=1$. Alle punkter inne i sirkelen – eller på sirkelperiferien vil tilhøre definisjonsmengden.

f)

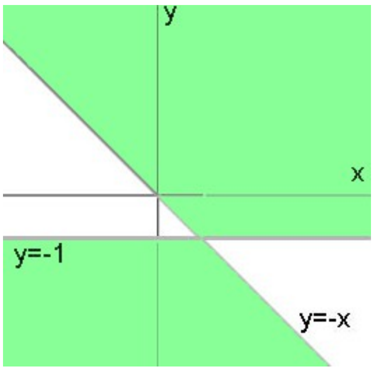
Argumentet til logaritmefunksjonen må være >0, $\frac{y+1}{y+x}>0$. Her kan vi først luke bort y=-1 som gir argument null, og y=-x som gir en verdi som ikke eksisterer. Deretter ser vi at kombinasjonen av positiv teller og positiv nevner og kombinasjonen av negativ teller og negativ nevner gir ok argument. Samler bitene , definisjonsmengden er begrenset av

$$y+1>0 \quad \wedge \quad y+x>0$$

$$y+1<0 \quad \wedge \quad y+x<0$$

Igjen lager vi en grafisk framstilling av definisjonsmengden, der $y+1=0$, altså den rette linja linja $y=-1$ er grensetilfellet for $y+1>0$ og $y+1<0$, og den rette linja $y=-x$ er grensetilfellet for $y+x>0$ og $y+x<0$.

Definisjonsmengden er altså de verdiene av x og y som ligger over $y=-1$ og samtidig til høyre for $y=-x$, eller under $y=-1$ og samtidig til venstre for $y=-x$.



7.10-2.abcdef

a)

$$\frac{\partial f}{\partial x}=[5y^6]\cdot 2\cdot x=10xy^6 \qquad \frac{\partial f}{\partial y}=[5x^2]\cdot 6\cdot y^5=30x^2y^5$$

b)

$$\frac{\partial f}{\partial x}=\frac{1}{2}(x^3+[3y^4])^{-\frac{1}{2}}\cdot 3x^2=\frac{3}{2}\frac{x^2}{\sqrt{x^3+3y^4}}$$

$$\frac{\partial f}{\partial x}=\frac{1}{2}([x^3]+3y^4)^{-\frac{1}{2}}\cdot 4\cdot 3y^3=6\frac{y^3}{\sqrt{x^3+3y^4}}$$

c)

$$\frac{\partial f}{\partial u}=2\cdot(-\sin(2u+[7v]))=-2\sin(2u+7v)$$

$$\frac{\partial f}{\partial v}=7\cdot(-\sin(2u+[7v]))=-7\sin(2u+7v)$$

d)

$$\frac{\partial f}{\partial u}=(1+\tan^2(u^2v))\cdot 2uv \qquad \frac{\partial f}{\partial v}=(1+\tan^2(u^2v))\cdot u^2$$

e)

$$\frac{\partial f}{\partial x}=(e^{3x+y}\cdot 3)\cdot(\ln(xy))+(e^{3x+y})\cdot(\frac{1}{xy}\cdot y)=e^{3x+y}(3\ln(xy)+\frac{1}{x})$$

$$\frac{\partial f}{\partial y}=(e^{3x+y}\cdot 1)\cdot(\ln(xy))+(e^{3x+y})\cdot(\frac{1}{xy}\cdot x)=e^{3x+y}(\ln(xy)+\frac{1}{y})$$

f)

$$\frac{\partial f}{\partial u}=(3\cdot\cos(3u+v^2))\cdot(\cos(u^2-2v))+(\sin(3u+v^2))\cdot(-(\sin(u^2-2v))\cdot 2u)=\ldots$$

$$\frac{\partial f}{\partial v}=(2v\cdot\cos(3u+v^2))\cdot(\cos(u^2-2v))+(\sin(3u+v^2))\cdot(-(\sin(u^2-2v))\cdot(-2))=\ldots$$

7.10- 3

$$\frac{\partial E}{\partial m}=\frac{1}{2}v^2\cdot\frac{d(m)}{dm}=\frac{1}{2}v^2$$

$$\frac{\partial E}{\partial v}=\frac{1}{2}m\cdot\frac{d(v^2)}{dv}=\frac{1}{2}m\cdot 2v=mv$$

7.10- 8

Bruker lineær approksimasjon,

Ny fynksjonsverdi: $f(a+h,b+k)\approx f(a,b)+f_x(a,b)h+f_y(a,b)k$

Avvik: $\Delta=f(a+h,b+k)-f(a,b)\approx f_x(a,b)h+f_y(a,b)k$

Maksimal feil: $6\cdot 1\cdot 0.1+1\cdot 8\cdot 0.1=0.14$

7.10- 15

$$\frac{\partial O}{\partial t}=\frac{\partial O}{\partial r}\cdot\frac{dr}{dt}+\frac{\partial O}{\partial h}\cdot\frac{dh}{dt}=2\pi(2r+h)\cdot\frac{dr}{dt}+2\pi r\cdot\frac{dh}{dt}$$
$$=2\pi(2\cdot 4+10)\cdot 3+2\pi 4\cdot(-2)=92\pi\ [\mathrm{cm}^2/\mathrm{s}]$$

7.11-1.acegi

a)

$$\frac{\partial f}{\partial x}=f_x=4\text{x}+0+4=4(x+1) \qquad \frac{\partial f}{\partial y}=f_y=0+2\text{y}+0+0=2\text{y}$$

$f_{xx}=4$, $f_{yy}=2$, $f_{xy}=0$

Mulig ekstremalverdi for (x, y) = (-1, 0)

$$\Delta=f_{xx}(a,b)\cdot f_{yy}(a,b)-(f_{xy}(a,b))^2=4\cdot 2-0^2=8 \quad , >0$$

Lokalt minimum for (-1, 0), $f(-1,0)=2(-1)^2+0^2+4(-1)-8=-10$

c)

$$\frac{\partial f}{\partial x}=f_x=2\text{x}-6=2(x-3) \qquad \frac{\partial f}{\partial y}=f_y=-6\text{y}-12=-6(y+2)$$

$f_{xx}=2$, $f_{yy}=-6$, $f_{xy}=0$

Mulig ekstremalverdi for (x, y) = (3, -2)

$$\Delta=f_{xx}(a,b)\cdot f_{yy}(a,b)-(f_{xy}(a,b))^2=2\cdot(-6)-0^2=-12 \quad , <0$$

(Sadelpunkt for (3, -2), $f(3,-2)=3^2-3\cdot(-2)^2-6\cdot 3-12\cdot(-2)-4=1$)

e)

$$\frac{\partial f}{\partial x}=f_x=2\text{x}-2\text{y}-6=2(x-y-3) \qquad \frac{\partial f}{\partial y}=f_y=-2\text{x}+4\text{y}+2=2(-x+2\text{y}+1)$$

$f_{xx}=2$, $f_{yy}=4$, $f_{xy}=-2$

Finner nullpunkter i deriverte,

$$\begin{aligned} x-y-3&=0 \\ -x+2\text{y}+1&=0 \end{aligned}$$ x=5 og y=2

Mulig ekstremalverdi for (x, y) = (5, 2)

$$\Delta=f_{xx}(a,b)\cdot f_{yy}(a,b)-(f_{xy}(a,b))^2=2\cdot 4-(-2)^2=4 \quad , >0$$

Lokalt minimum for (5, 2), $f(5,2)=5^2-2\cdot 5\cdot 2+2\cdot 2^2-6\cdot 5+2\cdot 2=-13$

g)

$$\frac{\partial f}{\partial x}=f_x=2\text{x}^2+12x=3\text{x}(x+4) \qquad \frac{\partial f}{\partial y}=f_y=-3\text{y}^2+12y=-3\text{y}(y-4)$$

$f_{xx}=4\text{x}+12$, $f_{yy}=-6\text{y}+12$, $f_{xy}=0$

Mulig ekstremalverdi for (x, y) = (0,0) eller (0,4) eller ((-4, 0) eller (-4, 4)

Tester (x,y)=(0,0), $f_{xx}=12$, $f_{yy}=12$

$$\Delta=f_{xx}(a,b)\cdot f_{yy}(a,b)-(f_{xy}(a,b))^2=12\cdot 12-0^2=144 \quad , >0$$

Lokalt maksimum for (0, 0), $f(0,0)=0^3-0^3+6\cdot 0^2+6\cdot 0^2=0$

Tester (x,y)=(0,4), $f_{xx}=12$, $f_{yy}=-12$

$$\Delta=f_{xx}(a,b)\cdot f_{yy}(a,b)-(f_{xy}(a,b))^2=12\cdot(-12)-0^2=-144 \quad , <0$$

(Sadelpunkt for (0,4)

Tester (x,y)=(-4,0), $f_{xx}=-4$, $f_{yy}=12$

$\Delta=f_{xx}(a,b)\cdot f_{yy}(a,b)-(f_{xy}(a,b))^2=-4\cdot 12-0^2=-48$, <0

(Sadelpunkt for (-4,0)

Tester (x,y)=(-4,4), $f_{xx}=-12$, $f_{yy}=-12$

$\Delta=f_{xx}(a,b)\cdot f_{yy}(a,b)-(f_{xy}(a,b))^2=(-12)\cdot(-12)-0^2=144$, >0

Lokalt maksimum for (0, 0), $f(0,0)=0^3-0^3+6\cdot 0^2+6\cdot 0^2=0$

i)

$\frac{\partial f}{\partial x}=f_x=3x^2-12y=3(x^2-4y)$ $\frac{\partial f}{\partial y}=f_y=-3y^2-12x=-3(y^2+4x)$

$f_{xx}=6x$, $f_{yy}=-6y$, $f_{xy}=-12$

Finner nullpunkter i deriverte,

$x^2-4y=0$
$y^2+4x=0$ x=0 og y=0 eller x=-4 og y=4 eller to komplekse løsninger.

Mulig ekstremalverdi for (x, y) = (0,0) eller (-4, 4)

Tester (x,y)=(0,0), $f_{xx}=0$, $f_{yy}=0$

$\Delta=f_{xx}(a,b)\cdot f_{yy}(a,b)-(f_{xy}(a,b))^2=0\cdot 0-(-12)^2=-144$, <0

(Sadelpunkt for (0,0)

Tester (x,y)=(-4,4), $f_{xx}=-24$, $f_{yy}=-24$

$\Delta=f_{xx}(a,b)\cdot f_{yy}(a,b)-(f_{xy}(a,b))^2=-24\cdot(-24)-(-12)^2=432$, >0

Lokalt maksimum for (-4, 4), $f(-4,4)=(-4)^3-4^3-12\cdot(-4)4+5=69$

13.1-2.abcde

a)

$a=0 \quad , \quad x-a=x$

$f(x)=x^2+4x+3 \quad , \quad f'(x)=2x+4 \quad , \quad f''(x)=2 \quad , \quad f'''(x)=0$

$f(0)=0+0+3=3 \quad , \quad f'(0)=0+4=4 \quad , \quad f''(0)=2$

Grad 1: $P_1(x)=f(0)+f'(0)(x-a)=3+4x$

Grad 2: $P_2(x)=f(0)+f'(0)(x-a)+\frac{f''(0)}{2!}(x-a)^2=3+4x+x^2$

Grad 3: $P_3(x)=f(0)+f'(0)(x-a)+\frac{f''(0)}{2!}(x-a)^2+\frac{f'''(0)}{3!}(x-a)^3=3+4x+x^2$

b)

$a=0 \quad , \quad x-a=x$

$f(x)=x^3-3x^2+5x-2 \quad , \quad f'(x)=3x^2-6x+5 \quad , \quad f''(x)=6x-6 \quad , \quad f'''(x)=6$

$f(0)=0+0+0-2=-2 \quad , \quad f'(0)=0+0+5=5 \quad , \quad f''(0)=0-6=-6 \quad , \quad f'''(0)=6$

Grad 1: $P_1(x)=f(0)+f'(0)(x-a)=-2+5x$

Grad 2: $P_2(x)=f(0)+f'(0)(x-a)+\frac{f''(0)}{2!}(x-a)^2=-2+5x-3x^2$

Grad 3: $P_3(x)=f(0)+f'(0)(x-a)+\frac{f''(0)}{2!}(x-a)^2+\frac{f'''(0)}{3!}(x-a)^3$

$=-2+5x-3x^2+x^3$

c)

$a=0 \quad , \quad x-a=x$

$f(x)=\tan x \quad , \quad f'(x)=1+\tan^2 x \quad , \quad f''(x)=2\tan x(1+\tan^2 x) \quad ,$

$f'''(x)=2(1+\tan^2 x)+6\tan^2 x(1+\tan^2 x)$

$f(0)=0 \quad , \quad f'(0)=1 \quad , \quad f''(0)=0 \quad , \quad f'''(0)=2$

Grad 1: $P_1(x)=f(0)+f'(0)(x-a)=0+x=x$

Grad 2: $P_2(x)=f(0)+f'(0)(x-a)+\frac{f''(0)}{2!}(x-a)^2=0+x+0=x$

Grad 3: $P_3(x)=f(0)+f'(0)(x-a)+\frac{f''(0)}{2!}(x-a)^2+\frac{f'''(0)}{3!}(x-a)^3$

$=0+x+0+\frac{2}{6}x^2=x+\frac{x^3}{3}$

d)

$a=0 \quad , \quad x-a=x$

$f(x)=\sqrt{4+x} \quad , \quad f'(x)=\frac{1}{2}(4+x)^{-\frac{1}{2}} \quad , \quad f''(x)=-\frac{1}{4}(4+x)^{-\frac{3}{2}} \quad , \quad f'''(x)=\frac{3}{8}(4+x)^{-\frac{5}{2}}$

$f(0)=2 \quad , \quad f'(0)=\frac{1}{4} \quad , \quad f''(0)=-\frac{1}{32} \quad , \quad f'''(0)=\frac{3}{256}$

Grad 1: $P_1(x)=f(0)+f'(0)(x-a)=2+\frac{1}{4}x$

Grad 2: $P_2(x)=f(0)+f'(0)(x-a)+\frac{f''(0)}{2!}(x-a)^2=2+\frac{1}{4}x-\frac{1}{64}x^2$

Grad 3:
$$P_3(x)=f(0)+f'(0)(x-a)+\frac{f''(0)}{2!}(x-a)^2+\frac{f'''(0)}{3!}(x-a)^3$$
$$=2+\frac{1}{4}x-\frac{1}{64}x^2+\frac{1}{512}x^3$$

e)

$a=0$, $x-a=x$

$f(x)=\sin^{-1}x$, $f'(x)=\frac{1}{2}(4+x)^{-\frac{1}{2}}$, $f''(x)=x(1-x^2)^{-\frac{3}{2}}$,

$f'''(x)=(1-x^2)^{-\frac{3}{2}}+3x^2(1-x^2)^{-\frac{5}{2}}$

$f(0)=0$, $f'(0)=1$, $f''(0)=0$, $f'''(0)=1$

Grad 1: $P_1(x)=f(0)+f'(0)(x-a)=x$

Grad 2: $P_2(x)=f(0)+f'(0)(x-a)+\frac{f''(0)}{2!}(x-a)^2=x$

Grad 3: $P_3(x)=f(0)+f'(0)(x-a)+\frac{f''(0)}{2!}(x-a)^2+\frac{f'''(0)}{3!}(x-a)^3=x+\frac{1}{6}x^3$

13.1-3.bcd

b)

$a=4$, $x-a=x-4$

$f(x)=\sqrt{x}=x^{\frac{1}{2}}$, $f'(x)=\frac{1}{2}x^{-\frac{1}{2}}$, $f''(x)=-\frac{1}{4}x^{-\frac{3}{2}}$, $f'''(x)=\frac{3}{8}x^{-\frac{5}{2}}$

$f(4)=2$, $f'(1)=\frac{1}{4}$, $f''(0)=-\frac{1}{32}$, $f'''(0)=\frac{3}{256}$

Grad 3:
$$P_3(x)=f(4)+f'(4)(x-4)+\frac{f''(4)}{2!}(x-4)^2+\frac{f'''(4)}{3!}(x-4)^3$$
$$=2+\frac{1}{4}(x-4)-\frac{1}{64}(x-4)^2+\frac{1}{512}(x-4)^3$$

c)

$a=1$, $x-a=x-1$

$f(x)=\frac{1}{x}$, $f'(x)=-x^{-2}$, $f''(x)=2x^{-3}$, $f'''(x)=-6x^{-4}$

$f(1)=1$, $f'(1)=-1$, $f''(1)=2$, $f'''(1)=-6$

Grad 3:
$$P_3(x)=f(1)+f'(1)(x-1)+\frac{f''(1)}{2!}(x-1)^2+\frac{f'''(1)}{3!}(x-1)^3$$
$$=1-(x-1)+(x-1)^2-(x-1)^3$$

d)

$a=\frac{\pi}{4}$, $x-a=x-\frac{\pi}{4}$

$f(x)=\frac{\cos x}{\sin x}$, $f'(x)=\frac{-\sin x\cdot\sin x-\cos x\cdot\cos x}{\sin^2 x}=-\frac{1}{\sin^2 x}=-(\sin x)^{-2}$,

$$f''(x)=-(-2)(\sin x)^{-3}\cdot\cos x=2\frac{\cos x}{(\sin x)^3} ,$$

$$f'''(x)=2\left[\frac{-\sin x\cdot(\sin x)^3-\cos x\cdot 3(\sin x)^2\cdot\cos x}{(\sin x)^6}\right]$$

$$=2\left[\frac{-(\sin x)^4-3(\sin x)^2\cdot(\cos x)^2}{(\sin x)^6}\right]$$

$$=-\frac{2}{(\sin x)^2}-6\frac{(\cos x)^2}{(\sin x)^4}$$

$f(\frac{\pi}{4})=1$, $f'(\frac{\pi}{4})=-(\frac{1}{\sqrt{2}})^{-2}=2$,

$$f''(\frac{\pi}{4})=2\frac{1/\sqrt{2}}{(1/\sqrt{2})^3}=4 ,$$

$$f'''(\frac{\pi}{4})=\frac{-2}{(1/\sqrt{2})^2}-6\frac{(1/\sqrt{2})^2}{(1/\sqrt{2})^4}=-16$$

Grad 3:

$$P_3(x)=f(1)+f'(\frac{\pi}{4})(x-1)+\frac{f''(\frac{\pi}{4})}{2!}(x-1)^2+\frac{f'''(\frac{\pi}{4})}{3!}(x-1)^3$$

$$=1-2(x-\frac{\pi}{4})+2(x-\frac{\pi}{4})^2-\frac{8}{3}(x-\frac{\pi}{4})^3$$

13.1-4.ab

a)

$f(a)=f(0)=b=1$ $\qquad x^2+5x-xy+2y=2$

1. orden derivert, $\quad 2x+5-y-xy'+2y'=0$

$$0+5-1-0+2y'=0 , \quad y'=-2$$

2. orden derivert, $\quad 2-y'-y'+xy''+2y''=0$

$$2--2--2+0+2y''=0 , \quad y''=-3$$

Grad 2: $\quad P_2(x)=f(0)+f'(0)(x-0)+\frac{f''(0)}{2!}(x-0)^2=1-2x-\frac{3}{2}x^2$

b)

$f(a)=f(0)=b=1$ $\qquad x^2y^2+2x^2+y^3=1$

1. orden derivert, $\quad 2xy^2+x^2 2yy'+4x+3y^2y'=0$

$$0+0+0+3y'=0 , \quad y'=0$$

2. orden derivert, $\quad 2y^2+2(2xyy'+x^2(y'y'+yy''))+4+3(2yy'y'+y^2y')=0$

$$2+0+4+3(0+0)=0 , \quad y''=-2$$

Grad 2: $$P_2(x)=f(0)+f'(0)(x-0)+\frac{f''(0)}{2!}(x-0)^2=1-x^2$$

13.1-5.abcd

a)

$a=0 \quad , \quad x-a=x$

$$f(x)=\frac{1}{2}(e^x+e^{-x}) \quad , \quad f'(x)=\frac{1}{2}(e^x-e^{-x}) \quad , \quad f''(x)=\frac{1}{2}(e^x+e^{-x}) \quad , \quad f'''(x)=\frac{1}{2}(e^x-e^{-x})$$

$$P_2(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2=\frac{2}{2}+0+\frac{2}{4}x^2=1+\frac{1}{2}x^2$$

$$R_2(x)=\frac{f'''(\mathbf{c})}{3!}(x-a)^3=\frac{e^c-e^{-c}}{12}x^3 \quad , \quad 0<c<x$$

b)

$a=0 \quad , \quad x-a=x$

$$f(x)=\tan^{-1}x \quad , \quad f'(x)=\frac{1}{x^2+1} \quad , \quad f''(x)=\frac{-2x}{(x^2+1)^2} \quad ,$$

$$f'''(x)=\frac{8x^2}{(x^2+1)^3}-\frac{2}{(x^2+1)^2}=\frac{8x^2}{(x^2+1)^3}-\frac{2(x^2+1)}{(x^2+1)^3}=2\frac{3x^2-1}{(x^2+1)^3}$$

$$P_2(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2=0+1x-0x^2=x$$

$$R_2(x)=\frac{f'''(\mathbf{c})}{3!}(x-a)^3=\frac{1}{3}\cdot\frac{3c^2-1}{(c^2+1)^3}x^3 \quad , \quad 0<c<x$$

c)

$a=0 \quad , \quad x-a=x$

$$f(x)=\ln(1+4x) \quad , \quad f'(x)=\frac{4}{4x+1} \quad , \quad f''(x)=\frac{-16}{(4x+1)^2} \quad , \quad f'''(x)=\frac{128}{(4x+1)^3}$$

$$P_2(x)=f(a)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2=0+4x-8x^2=4x-8x^2$$

$$R_2(x)=\frac{f'''(\mathbf{c})}{3!}(x-a)^3=\frac{64}{3}\cdot\frac{1}{(4c+1)^3}x^3 \quad , \quad 0<c<x$$

d)

$a=0 \quad , \quad x-a=x$

$$f(x)=e^{x^2} \quad , \quad f'(x)=2xe^{x^2} \quad , \quad f''(x)=2e^{x^2}+4x^2e^{x^2} \quad , \quad f'''(x)=12xe^{x^2}+8x^3e^{x^2}$$

$$P_2(x)=f(0)+f'(a)(x-a)+\frac{f''(a)}{2!}(x-a)^2=1+0+\frac{2}{2}x^2=1+x^2$$

$$R_2(x)=\frac{f'''(\mathbf{c})}{3!}(x-a)^3=\frac{12ce^{c^2}+8c^3e^{c^2}}{6}x^3=(2ce^{c^2}+\frac{4}{3}c^3e^{c^3})x^3 \quad , \quad 0<c<x$$

13.2-1.abcdefghijk

a)

$$a_n = 2 + \left(\frac{3}{5}\right)^n \qquad f(x) = 2 + \left(\frac{3}{5}\right)^x$$

$$\lim_{x\to\infty}\left(2+\left(\frac{3}{5}\right)^x\right) = \lim_{n\to\infty} 2 + \lim_{x\to\infty}\frac{1}{\left(\frac{5}{3}\right)^x} = 2$$

Konvergerer!

b)

$$a_n = 1 - \left(-\frac{3}{5}\right)^n \qquad f(x) = 1 - \left(-\frac{3}{5}\right)^x$$

$$\lim_{n\to\infty}\left(1-\left(-\frac{3}{5}\right)^x\right) = \lim_{n\to\infty} 1 - \lim_{n\to\infty}\frac{1}{\left(-\frac{5}{3}\right)^x} = 1$$

Konvergerer!

c)

$a_n = 2 + (-1)^n$, $a_n = \{\ldots, 3, 1, 3, 1, \ldots\}$ Divergerer!

d)

$$a_n = \frac{\sin n}{n^2+3} \qquad \frac{-1}{x^2+3} \le \frac{\sin x}{x^2+3} \le \frac{1}{x^2+3} \qquad \lim_{a_n\to\infty} a_n = 0$$

Konvergerer!

e)

$$a_n = \sin n \qquad \lim_{n\to\infty}\sin n = \lim_{n\to\infty}(\text{alle verdier fra } -1 \text{ til } 1, \neq 01) = ?!$$

Divergerer!

f)

$$a_n = 2^{\frac{1}{n}} = \sqrt[n]{2} \qquad \lim_{x\to\infty} 2^{\frac{1}{x}} = \lim_{M\to 0} 2^M = 1$$

Konvergerer!

g)

$$a_n = \frac{2^n}{n!} \qquad \lim_{n\to\infty}\frac{2\cdot 2\cdot 2\cdot 2\cdot\ldots\cdot 2}{1\cdot 2\cdot 3\cdot 4\cdot\ldots\cdot n} = 2\cdot\lim_{n\to\infty}\frac{1}{\frac{3}{2}}\cdot\frac{1}{\frac{4}{2}}\cdot\frac{1}{\frac{5}{2}}\cdot\ldots\cdot\frac{1}{\frac{n}{2}} = 0$$

Konvergerer!

h)

$$a_n = \ln\left(n + \frac{2}{n}\right) \qquad \lim_{x\to\infty}\ln\left(x+\frac{2}{x}\right) = \lim_{x\to\infty}\ln x$$

Divergerer!

i)

$$a_n = \frac{n^2+2n}{5n^3+4} \qquad \lim_{x\to\infty}\frac{x^2+2x}{5x^3+4} = \lim_{x\to\infty}\frac{\frac{1}{x}+\frac{2}{x^2}}{5+\frac{4}{x^3}} = \frac{0}{5} = 0$$

Konvergerer!

j)

$$a_n = \frac{3n^2-2}{4n^2+7n} \qquad \lim_{n\to\infty}\frac{3n^3-2}{4n^2+7n} = \lim_{x\to\infty}\frac{3-\frac{2}{x^3}}{\frac{4}{x}+\frac{7}{x^2}} = \frac{3}{'0'}$$

Divergerer!

k)

$$a_n=\frac{6n^2+5n-1}{n^2+2n+8} \qquad \lim_{x\to\infty}\frac{6x^2+5x-1}{x^2+2x+8}=\lim_{x\to\infty}\frac{6+\frac{5}{x}-\frac{1}{x^2}}{1+\frac{2}{x}+\frac{8}{x^2}}=6$$

Konvergerer!

13.2-5.abcd

a)

$$a_n=3^n \qquad S=\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}3^n$$

$$a_1=3^3=1 \ , \ a_2=3^2=9 \ , \ a_3=3^3=27$$

$$S_3=3+9+27=39$$

b)

$$a_n=\frac{n+2}{n+1} \qquad S=\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}\frac{n+2}{n+1}$$

$$a_1=\frac{3}{2} \ , \ a_2=\frac{4}{3} \ , \ a_3=\frac{5}{4}$$

$$S_3=\frac{3}{2}+\frac{4}{3}+\frac{5}{4}=\frac{49}{12}$$

c)

$$a_n=\frac{(-1)^{n-1}}{n(n+2)} \qquad S=\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n(n+2)}$$

$$a_1=\frac{1}{3} \ , \ a_2=-\frac{1}{8} \ , \ a_3=\frac{1}{15}$$

$$S_3=\frac{1}{3}-\frac{1}{8}+\frac{1}{15}=\frac{11}{40}$$

d)

$$a_n=(-1)^n\frac{n!}{(3n)!} \qquad S=\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}(-1)^n\frac{n!}{(3n)!}$$

$$a_1=-\frac{1}{6} \ , \ a_2=\frac{1}{360} \ , \ a_3=-\frac{1}{60480}$$

$$S_3=-\frac{1}{6}+\frac{1}{360}-\frac{1}{60480}=-\frac{9913}{60480}$$

13.2-6.abcd

a)

$$S=\frac{3}{2}+\frac{4}{3}+\frac{5}{4}+\ldots=\frac{2+1}{2}+\frac{3+1}{3}+\frac{4+1}{4}+\ldots$$

$$a_n=\frac{n+1}{n} \ , \quad n=2,3,4,\ldots$$

$$S=\sum_{n=2}^{\infty}a_n=\sum_{n=2}^{\infty}\frac{n+1}{n}$$

b)

$$1-\frac{1}{5}+\frac{1}{9}-\frac{1}{13}+\ldots=\frac{1}{0\cdot 4+1}-\frac{1}{1\cdot 4+1}+\frac{1}{2\cdot 4+1}-\frac{1}{3\cdot 4+1},\ldots$$

$$a_n=(-1)^n\frac{1}{4n+1} \quad , \quad n=0,1,2,\ldots$$

$$S=\sum_{n=0}^{\infty}a_n=\sum_{n=0}^{\infty}(-1)^n\frac{1}{4n+1}$$

c)

$$\frac{1}{6}+\frac{4}{7}+\frac{9}{8}+\frac{16}{9}+\frac{25}{10}+\ldots=\frac{1^2}{5+1}+\frac{2^2}{5+2}+\frac{3^2}{5+3}+\frac{4^2}{5+4}+\frac{5^2}{5+5}+\ldots$$

$$a_n=\frac{n^2}{5+n} \quad , \quad n=1,2,3,\ldots$$

$$S=\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}\frac{n^2}{5+n}$$

d)

$$\frac{5}{1\cdot 4}-\frac{7}{2\cdot 7}+\frac{9}{3\cdot 10}-\frac{11}{4\cdot 13}+\frac{13}{5\cdot 16}-\ldots=\frac{3+2\cdot 1}{1\cdot(3\cdot 1+1)}-\frac{3+2\cdot 2}{2\cdot(3\cdot 2+1)}+\frac{3+2\cdot 3}{3\cdot(3\cdot 3+1)}-\frac{3+2\cdot 4}{4\cdot(3\cdot 4+1)}+\ldots$$

$$a_n=(-1)^{n-1}\frac{3+2n}{n(3n+1)}$$

$$S=\sum_{n=1}^{\infty}a_n=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{3+2n}{n(3n+1)}$$

13.2-7.abcd

a)

$$a_1=1 \quad , \quad k=\frac{\frac{1}{5}}{1}=\frac{1}{5} \quad , \quad S=a_1\frac{1}{1-k}=1\frac{1}{1-\frac{1}{5}}=\frac{5}{4}$$

b)

$$a_1=3 \quad , \quad k=\frac{-1}{3}=-\frac{1}{3} \quad , \quad S=a_1\frac{1}{1-k}=3\frac{1}{1-(\frac{-1}{3})}=\frac{9}{4}$$

c)

$$a_1=2 \quad , \quad k=\frac{\frac{3}{2}}{2}=\frac{3}{4} \quad , \quad S=a_1\frac{1}{1-k}=2\frac{1}{1-\frac{3}{4}}=8$$

d)

$$a_1=\frac{1}{4} \quad , \quad k=\frac{-1/10}{1/4}=-\frac{2}{5} \quad , \quad S=a_1\frac{1}{1-k}=\frac{1}{4}\frac{1}{1-(\frac{-2}{5})}=\frac{5}{28}$$

13.2-8

$$a_3=\frac{3}{4} \quad , \quad a_5=\frac{1}{3}=k^4 a_1=k^2 a_3 \quad , \quad k=\sqrt{\frac{1/3}{3/4}}=\frac{2}{3} \quad , \quad a_1=\frac{1}{(\frac{2}{3})^4}\cdot\frac{1}{3}=\frac{27}{16}$$

$$S=a_1\frac{1}{1-k}=\frac{27}{16}\cdot\frac{1}{1-\frac{2}{3}}=\frac{81}{16}$$

13.2-9

$$a_6=\frac{5}{3} \quad , \quad a_9=-\frac{9}{25}=k^8 a_1=k^3 a_6 \quad , \quad k=\sqrt[3]{\frac{-9/25}{5/3}}=-\frac{3}{5} \quad , \quad a_1=\frac{1}{(-\frac{3}{5})^8}\cdot\frac{-9}{25}=-\frac{15625}{729}$$

$$S=a_1\frac{1}{1-k}=\frac{-15625}{729}\cdot\frac{1}{1-(-\frac{3}{5})}=-\frac{-78125}{5832}$$

13.2-10

$$a_1+a_2=a_1(1+k)=\frac{2}{3} \quad , \quad a_3-a_1=a_1(k^2-1)=-\frac{8}{9} \quad , \quad \frac{a_1(k^2-1)}{a_1(k+1)}=k-1=\frac{-8/9}{2/3}=-\frac{4}{3}$$

$$k=-\frac{1}{3} \qquad a_1(1+k)=a_1(1-\frac{1}{3})=\frac{2}{3} \quad , \quad a_1=1$$

$$S=a_1\frac{1}{1-k}=1\cdot\frac{1}{1-(-\frac{1}{3})}=\frac{3}{4}$$

13.2-11.abcde

a)

$$a_n=x^n \quad , \quad n=1,2,3,\ldots \qquad k=\frac{a_{n+1}}{a_n}=\frac{x^{n+1}}{x^n}=x$$

Rekka er konvergent fordi $k=x$ og $|k|=|x|<1$,

$$S=a_1\frac{1}{1-k}=x\frac{1}{1-x}$$

b)

$$a_n=(-1)^{n-1}x^{2(n-1)} \quad , \quad n=1,2,3,\ldots \qquad k=\frac{a_{n+1}}{a_n}=\frac{(-1)^{n+1-1}x^{2(n+1-1)}}{(-1)^{n-1}x^{2(n-1)}}=-x^2$$

Rekka er konvergent fordi $k=-x^2$ og $|k|=|-x^2|<1$,

$$S=a_1\frac{1}{1-k}=1\frac{1}{1-(-x^2)}=\frac{1}{1+x^2}$$

c)

$$z_n=8^{n-1}\cdot z^{3n-1} \quad , \quad n=1,2,3,\ldots \qquad k=\frac{a_{n+1}}{a_n}=\frac{8^{n+1-1}\cdot z^{3(n+1)-1}}{8^{n-1}\cdot z^{3n-1}}=8z^3$$

Rekka er konvergent fordi $k=8z^3$ og $|k|=8|z^3|< 8\cdot(\frac{1}{2})^3=1$

$S=a_1\frac{1}{1-k}=z^2\frac{1}{1-8z^3}$ 0 over {2 x + 3}

d)

$a_n=(-1)^{n-1}\cdot e^{-2(n-1)y}$, $n=1,2,3,...$ $k=\frac{a_{n+1}}{a_n}=\frac{(-1)^{n+1-1}\cdot e^{-2(n+1-1)y}}{(-1)^{n-1}\cdot e^{-2(n-1)y}}=-e^{-2y}$

Rekka er konvergent fordi $k=-e^{-2y}$ og $|k|=\frac{1}{(e^y)^2}< 1$ for $y>0$

$S=a_1\frac{1}{1-k}=1\frac{1}{1-(-e^{-2y})}=\frac{1}{1+e^{-2y}}=\frac{e^{2y}}{e^{2y}+1}$

e)

$a_n=(-1)^{n-1}\cdot(t-2)^{n-1}$, $n=1,2,3,...$ $k=\frac{a_{n+1}}{a_n}=\frac{(-1)^{n+1-1}\cdot(t-2)^{n+1-1}}{(-1)^{n-1}\cdot(t-2)^{n-1}}=-(t-2)$

Er $k>-1$? $-(t-2)>-1$ >> $-t+2>-1$ >> $-t>-3$ >> $t<3$ JA

Er $k<1$? $-(t-2)<1$ >> $-t+2<1$ >> $-t<-1$ >> $t>3$ JA

$S=a_1\frac{1}{1-k}=1\frac{1}{1-(-(t-2))}=\frac{1}{t-1}$

13.2-12

Rekkeledd $a_1=2x+3$ $a_2=k\,a_1=4x-1$

Kvotient $k=\frac{a_2}{a_1}=\frac{4x-1}{2x+3}$

Konvergens, $|k|<1$ $-1<\frac{4x-1}{2x+3}<1$

Ulikhet 1 $-1<\frac{4x-1}{2x+3}$ => $0<1+\frac{4x-1}{2x+3}$

=> $0<\frac{2x+3}{2x+3}+\frac{4x-1}{2x+3}$ => $0<\frac{6x+2}{2x+3}$

Tallinjedrøfting

```
_ _ _ _ _ _ _ _ _ -1/3___________
_ _ _ _ -3/2_____________________
_________ -3/2_ _ -1/3___________ *)
```

Ulikhet 2 $\frac{4x-1}{2x+3}<1$ => $\frac{4x-1}{2x+3}-1<0$

=> $\frac{4x-1}{2x+3}-\frac{2x+3}{2x+3}<0$ => $\frac{-2x+4}{2x+3}>0$

Tallinjedrøfting

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___________________________2 _ _
_ _ _ _-3/2______________________
_ _ _ _-3/2_______________2 _ _    **)
```

Konvergensområde, begge ulikheter oppfylt for $-\frac{1}{3}<x<2$

Rekkesum $s=\frac{a_1}{1-k}=\frac{2x+3}{1-\frac{4x-1}{2x+3}}=\frac{(2x+3)^2}{4-2x}$

13.2-16

Når ballen er høyest før det *n*-te sprett er høyden y_n ,

$y_1=4$, $y_2=0.6\cdot y_{1,}...$, $y_n=0.6\cdot y_{n-1}$

Når ballen spretter opp fra gulvet etter at den har vært i høyde y_n vil den bevege seg $2\cdot y_{n+1}$ før den igjen spretter fra gulvet. Ballen faller 4 m før første sprett.

Total distanse, $S=y_1+2\cdot y_2+2\cdot y_3+...=y_1+2\cdot 0.6\,y_1+2\cdot 0.6^2\,y_1+...$

Summen deles opp for å få en geometrisk rekke,

$$S=y_1+2(0.6\cdot y_1+0.6^2\,y_1+...)=4+2(0.6\cdot 4\frac{1}{1-0.6})=16\ [\mathrm{m}]$$

13.3-1.abcdfg

a)

Fra teoremet: $e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+...+\frac{x^n}{n!}+...$

Om vi substituerer $x\rightarrow -2x$ får vi

$$e^{-2x}=\sum_{n=0}^{\infty}\frac{(-2x)^n}{n!}=\sum_{n=0}^{\infty}\frac{(-2)^n x^n}{n!}=\sum_{n=0}^{\infty}(-1)^n\frac{2^n}{n!}x^n=1-\frac{2}{1!}x+\frac{2^2}{2!}x^2-...$$

b)

Fra teoremet: $e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+...+\frac{x^n}{n!}+...$

Om vi substituerer $x\rightarrow x^2$ får vi

$$e^{x^2}=\sum_{n=0}^{\infty}\frac{(x^2)^n}{n!}=\sum_{n=0}^{\infty}\frac{x^{2n}}{n!}=1+\frac{1}{1!}x^2+\frac{1}{2!}x^4+...$$

c)

Fra teoremet: $e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+...+\frac{x^n}{n!}+...$

Om vi substituerer $x\rightarrow -x$ og multipliserer på hver side med x^2 får vi

$$x^2\cdot e^{-x}=x^2\cdot\sum_{n=0}^{\infty}\frac{(-x)^n}{n!}=\sum_{n=0}^{\infty}x^2\frac{(-1)^n x^n}{n!}=\sum_{n=0}^{\infty}(-1)^n\frac{x^{n+2}}{n!}=x^2-\frac{x^3}{1!}+\frac{x^4}{2!}-...$$

d)

Fra teoremet: $e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+...+\frac{x^n}{n!}+...$

Om vi multipliserer på hver side med $1+2x$ får vi

$$(1+2x)\cdot e^{-x}=(1+2x)\cdot\sum_{n=0}^{\infty}\frac{x^n}{n!}=\sum_{n=0}^{\infty}\frac{x^n}{n!} + 2x\sum_{n=0}^{\infty}\frac{x^n}{n!}$$
$$=(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^n}{n!}+\ldots) + 2x(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^n}{n!}+\ldots)$$
$$=1+x+2x+\frac{x^2}{2!}+\frac{4x^2}{2!}+\frac{x^3}{3!}+\frac{6x^3}{3!}+\ldots=1+\frac{3x}{1!}+\frac{5x^2}{2!}+\frac{7x^3}{3!}+\ldots=\sum_{n=0}^{\infty}\frac{(2n+1)x^n}{n!}$$

f)

Fra teoremet: $\sin x=x-\frac{1}{3!}x^3+\frac{1}{5!}x^5-\frac{1}{7!}x^7+\ldots=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+1}$ for alle x

Om vi substituerer $x\to x^2$ og multipliserer på hver side med x^2 får vi

$$x^2\sin(x^2)=x^2\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}(x^2)^{2n+1}=\sum_{n=0}^{\infty}x^2\frac{(-1)^n}{(2n+1)!}x^{4n+2}=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{4n+4}$$

g)

Fra teoremet: $\cos x=1-\frac{x^1}{2!}x^2+\frac{1}{4!}x^4-\frac{1}{6!}x^6+\ldots=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n}$ for alle x

Om vi substituerer $x\to\frac{x}{3}$ og multipliserer på hver side med x får vi

$$x\cos(\frac{x}{3})=x\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}(\frac{x}{3})^{2n}=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}\cdot\frac{1}{3^{2n}}x\cdot x^{2n}=\sum_{n=0}^{\infty}\frac{(-1)^n}{3^{2n}(2n)!}\cdot x^{2n+1}$$

13.3-2.abd

a)

$$f(x)=3^x=(e^{\ln 3})^x=e^{x\ln 3}$$
$$f'(x)=\ln 3\, e^{x\ln 3}\quad,\quad f''(x)=(\ln 3)^2e^{x\ln 3}\quad,\quad f'''(x)=(\ln 3)^3e^{x\ln 3}$$
$$f(0)=1\quad,\quad f'(0)=\ln 3\quad,\quad f''(0)=(\ln 3)^2\quad,\quad f'''(0)=(\ln 3)^3$$
$$3^x=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\ldots=1+(\ln 3)x+\frac{(\ln 3)^2}{2!}x^2+\frac{(\ln 3)^3}{3!}x^3+\ldots$$

b)

$$f(x)=\sin^2 x$$
$$f'(x)=2\cos x\cdot\sin x\quad,\quad f''(x)=2\cos^2 x-2\sin^2 x\quad,\quad f'''(x)=-8\cos x\cdot\sin x$$
$$f^{(4)}(x)=8\sin^2 x-8\cos^2 x\quad,\quad f^{(5)}(x)=8\cos x\cdot\sin x\quad,\quad f^{(6)}(x)=32\cos^2 x-32\sin^2 x$$
$$f(0)=0\quad,\quad f'(0)=0\quad,\quad f''(0)=2\quad,\quad f'''(0)=0\quad,$$
$$f^{(4)}(0)=-8\quad,\quad f^{(5)}(0)=0\quad,\quad f^{(6)}(0)=32$$
$$\sin^2 x=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\ldots=0+0+\frac{2}{2!}x^2+0-\frac{8}{4!}x^4+0+\frac{32}{6!}x^6-\ldots=x^2-\frac{2^3}{4!}+\frac{2^5}{6!}x^6-\ldots$$

d)

$$f(x)=\ln(\cos x)$$
$$f'(x)=-\tan x\quad,\quad f''(x)=-1-\tan^2 x\quad,\quad f'''(x)=-2\tan x-2\tan^2 x$$

$$f^{(4)}(x)=-2-8\tan^2 x-6\tan^4 x$$

$$f(0)=0 \quad , \quad f'(0)=0 \quad , \quad f''(0)=-1 \quad , \quad f'''(0)=0 \quad , \quad f^{(4)}(0)=-2$$

$$\ln\cos x=f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\ldots=0+0-\frac{1}{2!}x^2+0-\frac{2}{4!}x^4-\ldots=-\frac{1}{2}x^2-\frac{1}{12}x^4-\ldots$$

13.3-3.abc

a)

Taylorrekka til $f(x)=e^x$ rundt $a=3$ er

$$e^x=c_0+c_1(x-3)+c_2(x-3)^2+c_3(x-3)^3+\ldots$$

der $c_0=f(3)=e^3 \quad , \quad c_1=f'(3)=e^3 \quad , \quad c_2=\frac{f''(3)}{2!}=\frac{e^3}{2!} \quad , \quad c_3=\frac{f'''(3)}{3!}=\frac{e^3}{3!} \quad , \ldots$

altså, $e^x=e^3+e^3(x-3)+\frac{e^3}{2!}(x-3)^2+\frac{e^3}{3!}(x-3)^3+\ldots=e^3\left(1+(x-3)+\frac{1}{2!}(x-3)^2+\frac{1}{3!}(x-3)^3+\ldots\right)$

b)

Taylorrekka til $f(x)=\sin x$ rundt $a=\frac{\pi}{4}$ er

$$\sin x=c_0+c_1\left(x-\frac{\pi}{4}\right)+c_2\left(x-\frac{\pi}{4}\right)^2+c_3\left(x-\frac{\pi}{4}\right)^3+\ldots$$

der $c_0=f\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2} \quad , \quad c_1=f'\left(\frac{\pi}{4}\right)=\cos\frac{\pi}{4}=\frac{\sqrt{2}}{2} \quad , \quad c_2=\frac{f''\left(\frac{\pi}{4}\right)}{2!}=\frac{-\sin\frac{\pi}{4}}{2!}=\frac{-\sqrt{2}}{2\cdot 2!} \quad ,$

$c_3=\frac{f'''\left(\frac{\pi}{4}\right)}{3!}=\frac{-\cos\frac{\pi}{4}}{3!}=-\frac{\sqrt{2}}{2\cdot 3!} \quad , \ldots$

altså, $\sin x=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)-\frac{\sqrt{2}}{2\cdot 2!}\left(x-\frac{\pi}{4}\right)^2-\frac{\sqrt{2}}{2\cdot 3!}\left(x-\frac{\pi}{4}\right)^3+\ldots$

$$=\frac{\sqrt{2}}{2}\left(1+\left(x-\frac{\pi}{4}\right)-\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2-\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+\ldots\right)$$

c)

Taylorrekka til $f(x)=\cos x$ rundt $a=\frac{\pi}{4}$ er

$$\cos x=c_0+c_1\left(x-\frac{\pi}{4}\right)+c_2\left(x-\frac{\pi}{4}\right)^2+c_3\left(x-\frac{\pi}{4}\right)^3+\ldots$$

der $c_0=f\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2} \quad , \quad c_1=f'\left(\frac{\pi}{4}\right)=-\sin\frac{\pi}{4}=-\frac{\sqrt{2}}{2} \quad , \quad c_2=\frac{f''\left(\frac{\pi}{4}\right)}{2!}=\frac{-\cos\frac{\pi}{4}}{2!}=\frac{-\sqrt{2}}{2\cdot 2!} \quad ,$

$c_3=\frac{f'''\left(\frac{\pi}{4}\right)}{3!}=\frac{\sin\frac{\pi}{4}}{3!}=-\frac{\sqrt{2}}{2\cdot 3!} \quad , \ldots$

altså, $\cos x=\frac{\sqrt{2}}{2}-\frac{\sqrt{2}}{2}\left(x-\frac{\pi}{4}\right)-\frac{\sqrt{2}}{2\cdot 2!}\left(x-\frac{\pi}{4}\right)^2+\frac{\sqrt{2}}{2\cdot 3!}\left(x-\frac{\pi}{4}\right)^3+\ldots$

$$=\frac{\sqrt{2}}{2}\left(1-\left(x-\frac{\pi}{4}\right)-\frac{1}{2!}\left(x-\frac{\pi}{4}\right)^2+\frac{1}{3!}\left(x-\frac{\pi}{4}\right)^3+\ldots\right)$$

13.3-4.abc

a)

Taylorrekka til $f(x)=\dfrac{1}{(1+x)^2}$ rundt $a=0$ er

$$\cos x=c_0+c_1(x-0)+c_2(x-0)^2+c_3(x-0)^3+...$$

der $c_0=f(0)=1$, $c_1=f'(0)=\dfrac{-2}{(1+0)^3}=-2$, $c_2=\dfrac{f''(0)}{2!}=\dfrac{\frac{6}{(1+0)^4}}{2!}=3$,

$c_3=\dfrac{f'''(0)}{3!}=\dfrac{\frac{-24}{(1+0)^5}}{3!}=-4$, ...

altså, $\dfrac{1}{(1+x)^2}=1-2x+3x^2-4x+...=\sum_{n=0}^{\infty}(-1)^n(n+1)x^n$

b)

Taylorrekka til $f(x)=e^x$ rundt $a=-2$ er

$$e^x=c_0+c_1(x+2)+c_2(x+2)^2+c_3(x+2)^3+...$$

der $c_0=f(-2)=e^{-2}$, $c_1=f'(-2)=e^{-2}$, $c_2=\dfrac{f''(-2)}{2!}=\dfrac{e^{-2}}{2!}$,

$c_3=\dfrac{f'''(-2)}{3!}=\dfrac{e^{-2}}{3!}$

altså, $e^x=e^{-2}\left(1+(x+2)+\dfrac{1}{2!}(x+2)^2+\dfrac{1}{3!}(x+2)^3+...\right)=e^{-2}\sum_{n=0}^{\infty}\dfrac{1}{n!}(x+2)^n$

c)

Taylorrekka til $f(x)=\dfrac{1}{2-x}$ rundt $a=1$ er

$$e^x=c_0+c_1(x+2)+c_2(x+2)^2+c_3(x+2)^3+...$$

der $c_0=f(1)=1$, $c_1=f'(1)=\dfrac{1}{(2-1)^2}=1$, $c_2=\dfrac{f''(1)}{2!}=\dfrac{\frac{2}{(2-1)^3}}{2!}=\dfrac{2}{2!}=1$,

$c_3=\dfrac{f'''(1)}{3!}=\dfrac{\frac{6}{(2-1)^4}}{3!}=\dfrac{6}{3!}=1$

altså, $\dfrac{1}{2-x}=1+(x-1)+(x-1)^2+(x-1)^3+...=\sum_{n=0}^{\infty}(x-1)^n$

13.3-5.cdefg

c)

Kombinerer $e^x=\sum_{n=0}^{\infty}\dfrac{x^n}{n!}=1+x+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+...+\dfrac{x^n}{n!}+...$

og $$x\,e^{-x}=x\sum_{n=0}^{\infty}\frac{(-x)^n}{n!}=x\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\ldots\pm\frac{x^n}{n!}+\ldots\right)=x-x^2+\frac{x^3}{2!}-\frac{x^4}{3!}+\ldots\pm\frac{x^{n+1}}{n!}+\ldots$$

til $$e^x-x\,e^{-x}=1+\left(\frac{1}{2!}+1\right)x^2+\left(\frac{1}{3!}-\frac{1}{2!}\right)x^3+\left(\frac{1}{4!}+\frac{1}{3!}\right)x^4+\ldots+\left(\frac{1}{n!}\pm\frac{1}{(n-1)!}\right)x^n+\ldots$$

$$=1+\left(\frac{1}{2!}+\frac{2}{2!}\right)x^2+\left(\frac{1}{3!}-\frac{3}{3!}\right)x^3+\left(\frac{1}{4!}+\frac{4}{4!}\right)x^4+\ldots+\left(\frac{1}{n!}\pm\frac{n}{n!}\right)x^n+\ldots$$

$$e^x-x\,e^{-x}=\sum_{n=0}^{\infty}\frac{1+n(-1)^n}{n!}x^n$$

d)

Utvider $$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^n}{n!}+\ldots$$

til $$e^{3x}=\sum_{n=0}^{\infty}\frac{(3x)^n}{n!}=1+3x+\frac{3^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{3^4x^4}{4!}+\frac{3^5x^5}{5!}+\ldots+\frac{3^nx^n}{n!}+\ldots$$

og $$x^2e^{3x}=x^2\sum_{n=0}^{\infty}\frac{(3x)^n}{n!}=x^2+3x^3+\frac{3^2x^4}{2!}+\frac{3^3x^5}{3!}+\ldots+\frac{3^nx^{n+2}}{n!}+\ldots$$

kombinerer

$$(x^2+1)e^{3x}=1+3x+\left(\frac{3^2}{2!}+1\right)x^2+\left(\frac{3^3}{3!}+3\right)x^3+\left(\frac{3^4}{4!}+\frac{3^2}{2!}\right)x^4+\left(\frac{3^5}{5!}+\frac{3^3}{3!}\right)x^5+\ldots$$

$$=1+3x+\ldots+\left(\frac{3^n}{n!}+\frac{3^{n-2}}{(n-2)!}\right)x^n+\ldots$$

$$=1+3x+\ldots+\left(\frac{3^n}{n!}+\frac{(n-1)n\,3^{n-2}}{n!}\right)x^n+\ldots=1+3x+\ldots+\frac{3^n+(n-1)n\,3^{n-2}}{n!}x^n+\ldots$$

$$=1+3x+\ldots+\frac{9+(n-1)n}{n!}3^{n-2}x^n+\ldots=9\cdot3^{-2}x^0+9\cdot3^{-1}x^1+\ldots+\frac{9+(n-1)n}{n!}3^{n-2}x^n+\ldots$$

$$=\sum_{n=0}^{\infty}\frac{9+(n-1)n}{n!}3^{n-2}x^n$$

e)

Utvider $$e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^n}{n!}+\ldots$$

til $$x\,e^x=x\sum_{n=0}^{\infty}\frac{(x)^n}{n!}=x+x^2+\frac{x^3}{2!}+\frac{x^4}{3!}+\frac{x^5}{4!}+\ldots+\frac{x^{n+1}}{n!}+\ldots$$

og $$2e^x=2\sum_{n=0}^{\infty}\frac{(x)^n}{n!}=2+2x+\frac{2x^2}{2!}+\frac{2x^3}{3!}+\frac{2x^4}{4!}+\ldots+\frac{2x^n}{n!}+\ldots$$

og $$x^2e^{-x}=x^2\sum_{n=0}^{\infty}\frac{(-x)^n}{n!}=x^2-x^3+\frac{x^4}{2!}-\frac{x^5}{3!}+\ldots\pm\frac{x^{n+2}}{n!}+\ldots$$

Samler bitene

$$(x+2)e^x+x^2e^{-x}=\underbrace{x+2+x^2}_{n=0}+\underbrace{x^2+2x-x^3}_{n=1}+\underbrace{\frac{x^3}{2!}+\frac{2x^2}{2!}+\frac{x^4}{2!}}_{n=2}+\frac{x^4}{3!}+\frac{2x^3}{3!}-\frac{x^5}{3!}+\frac{x^5}{4!}+\frac{2x^4}{4!}+\frac{x^6}{4!}+...$$

Samler ledd etter grad av x ,

$$=2x^0+3x^1+(1+1+\frac{2}{2!})x^2+(-1+\frac{1}{2!}+\frac{2}{3!})x^3+(\frac{1}{2!}+\frac{1}{3!}+\frac{2}{4!})x^4+(-\frac{1}{3!}+\frac{1}{4!}-\frac{2}{5!})x^5+...$$

$$=(0+0+\frac{2}{0!})x^0+(0+\frac{1}{0!}+\frac{2}{1!})x^1+(-\frac{1}{1!}+\frac{1}{2!}+\frac{2}{3!})x^2+(\frac{1}{2!}+\frac{1}{3!}+\frac{2}{4!})x^3+(-\frac{1}{3!}+\frac{1}{4!}-\frac{2}{5!})x^4+...$$

$$(\pm\frac{1}{(n-2)!}+\frac{1}{(n-1)!}+\frac{2}{n!})x^n+...$$

Det generelle leddet kan skrives om til

$$(\pm\frac{(n-1)n}{n!}+\frac{n}{n!}+\frac{2}{n!})x^n+...$$

- og rekka kan uttrykkes som

$$\sum_{n=0}^{\infty}\frac{n+2+(-1)^n n(n-1)}{n!}x^n$$

f)

Vi har at $\sin x=x-\frac{1}{3!}x^3+\frac{1}{5!}x^5-\frac{1}{7!}x^7+...=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+1}$

$$\cos x=1-\frac{1}{2!}x^2+\frac{1}{4!}x^4-\frac{1}{6!}x^6+...=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n}$$

som gir $3\sin x=3x-\frac{3}{3!}x^3+\frac{3}{5!}x^5-\frac{3}{7!}x^7+...$

$$x\cos x=x-\frac{1}{2!}x^3+\frac{1}{4!}x^5-\frac{1}{6!}x^7+...$$

Summerer

$$3\sin x+x+x\cos x=3x+x-\frac{3}{3!}x^3-\frac{1}{2!}x^3+\frac{3}{5!}x^5+\frac{1}{4!}x^5-\frac{3}{7!}x^7-\frac{1}{6!}x^7+...$$

$$=\frac{3+1}{1!}x^1-(\frac{3}{3!}+\frac{3}{3!})x^3+(\frac{3}{5!}+\frac{5}{5!})x^5-(\frac{3}{7!}+\frac{7}{7!})x^7+...\pm\frac{3+(2n+1)}{(2n+1)!}x^{2n+1}+...$$

$$=\sum_{n=0}^{\infty}(-1)^n\frac{4n+2}{(2n+1)!}x^{2n+1}$$

($2n+1$, $n=0,1,2,3,...$ lager tallfølgen 1, 3, 5, 7, ...)

g)

Vi har at $\sin x=x-\frac{1}{3!}x^3+\frac{1}{5!}x^5-\frac{1}{7!}x^7+...=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+1}$

$$\cos x=1-\frac{1}{2!}x^2+\frac{1}{4!}x^4-\frac{1}{6!}x^6+...=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n}$$

som gir $x^2\sin 2x=x^2(2x)-x^2\frac{1}{3!}(2x)^3+x^2\frac{1}{5!}(2x)^5-\frac{1}{7!}(2x)^7+...$

$$3x\cos 2x=3x-3x\frac{1}{2!}(2x)^2+3x\frac{1}{4!}(2x)^4-3x\frac{1}{6!}(2x)^6+...$$

Summerer

$$x^2\sin 2x+3x\cos 2x=3x+2^1x^{2+1}-3\frac{2^2}{2!}x^{2+1}-\frac{2^3}{3!}x^{2+3}+3\frac{2^4}{4!}x^{4+1}+\frac{2^5}{5!}x^{5+2}-3\frac{2^6}{6!}x^{6+1}+\dots$$
$$=3x+\frac{2\cdot 2^1-3\cdot 2^3}{2!}x^{2+1}-\frac{4\cdot 2^3-3\cdot 2^4}{4!}x^{4+1}+\frac{6\cdot 2^5-3\cdot 2^6}{6!}x^{6+1}+\dots+\frac{2n\cdot 2^{2n-1}-3\cdot 2^{2n}}{(2n)!}x^{2n+1}+\dots$$
$$=\sum_{n=0}^{\infty}(-1)^{n-1}\frac{(n-3)2^{2n}}{(2n)!}x^{2n+1}$$

($2n1$, $n=0,1,2,3,\dots$ lager tallfølgen 0, 2, 4, 6, ...)

13.3-6.abc

a)

$$\ln(1+x)^4=4\cdot\ln(1+x)=4(x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+\dots)=4\cdot\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{n}x^n$$

b)

$$\ln(1+x^3)=x^3-\frac{1}{2}(x^3)^2+\frac{1}{3}(x^3)^3-\frac{1}{4}(x^3)^4+\dots=\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{n}(x^3)^n=\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{n}x^{3n}$$

c)

Omformer litt på funksjonsuttrykket, $f(x)=\ln(5+x)=\ln(5(1+\frac{x}{5}))=\ln 5+\ln(1+\frac{x}{5})$

og bytter x med $\frac{5}{x}$ i rekkeutviklingen for $\ln(1+x)$,

$$\ln(5+x)=\ln 5+\frac{x}{5}-\frac{1}{2}(\frac{x}{5})^2+\frac{1}{3}(\frac{x}{5})^3-\frac{1}{4}(\frac{x}{5})^4+\dots\pm\frac{1}{n5^n}x^n+\dots$$
$$=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n5^n}x^n$$

13.3-7.abc

a)

Taylorrekka til $f(x)=\ln x$ rundt $a=1$ er

$$\ln x=c_0+c_1(x-1)+c_2(x-1)^2+c_3(x-1)^3+\dots$$

der $c_0=f(1)=0$, $c_1=f'(1)=\frac{1}{1}=1$, $c_2=\frac{f''(1)}{2!}=\frac{-1/1^2}{2!}=\frac{-1}{2!}=\frac{-1}{2}$,

$$c_3=\frac{f'''(1)}{3!}=\frac{2/1^3}{3!}=\frac{2}{3!}=\frac{1}{3} \quad , \quad c_4=\frac{f^{(4)}(1)}{4!}=\frac{-6/1^4}{4!}=\frac{-6}{4!}=\frac{-1}{4}$$

altså, $\ln x=0+1(x-1)-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3-\frac{1}{4}(x-1)^4+\dots=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n}(x-1)^n$

b)

Taylorrekka til $f(x)=\ln x^2$ rundt $a=2$ er

$$\ln x^2=c_0+c_1(x-2)+c_2(x-2)^2+c_3(x-2)^3+\dots$$

der $\quad c_0 = f(2) = 2\ln 2 \quad , \quad c_1 = f'(1) = 2/2 = 1 \quad , \quad c_2 = \dfrac{f''(1)}{2!} = \dfrac{-2/2^2}{2!} = \dfrac{-1}{4} \quad ,$

$$c_3 = \frac{f'''(2)}{3!} = \frac{4/2^3}{3!} = \frac{1}{12} \quad , \quad c_4 = \frac{f^{(4)}(2)}{4!} = \frac{-12/2^4}{4!} = \frac{-1}{32} \quad , \quad c_5 = \frac{f^{(5)}(2)}{5!} = \frac{48/2^5}{5!} = \frac{1}{80}$$

altså, $\quad \ln x^2 = 2\ln 2 + \dfrac{1}{1\cdot 2^0}(x-2) - \dfrac{1}{2\cdot 2^1}(x-2)^2 + \dfrac{1}{3\cdot 2^2}(x-2)^3 - \dfrac{1}{4\cdot 2^3}(x-2)^4 + \dfrac{1}{5\cdot 2^4}(x-2)^5 + \ldots$

$$= 2\ln 2 + \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n\cdot 2^{n-1}} (x-1)^n$$

c)

Taylorrekka til $\quad f(x) = \ln\sqrt[3]{x} = \dfrac{1}{3}\ln x \quad$ rundt $\quad a = 3 \quad$ er

$$\ln\sqrt[3]{x} = c_0 + c_1(x-2) + c_2(x-2)^2 + c_3(x-2)^3 + \ldots$$

der $\quad c_0 = f(3) = \dfrac{1}{3}\ln 3 \quad , \quad c_1 = f'(3) = 1/(3\cdot 3) = \dfrac{1}{9} \quad , \quad c_2 = \dfrac{f''(3)}{2!} = \dfrac{-1/(3\cdot 3^2)}{2!} = \dfrac{-1}{27\cdot 2} \quad ,$

$$c_3 = \frac{f'''(3)}{3!} = \frac{2/(3\cdot 3^3)}{3!} = \frac{1}{81\cdot 3} \quad , \quad c_4 = \frac{f^{(4)}(1)}{4!} = \frac{-2/3^4}{4!} = \frac{-1}{243\cdot 4} \quad ,$$

$$c_5 = \frac{f^{(5)}(3)}{5!} = \frac{8/3^5}{5!} = \frac{1}{729\cdot 5}$$

altså, $\quad \ln\sqrt[3]{x} = \dfrac{1}{3}\ln 3 + \dfrac{1}{1\cdot 3^2}(x-3) - \dfrac{1}{2\cdot 3^3}(x-3)^2 + \dfrac{1}{3\cdot 3^3}(x-3)^3 + \dfrac{1}{4\cdot 3^4}(x-3)^4 + \ldots$

$$= \frac{1}{3}\ln 3 + \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n\cdot 3^{n+1}} (x-3)^n$$

13.3-8

a) $\quad \dbinom{6}{3} = \dfrac{6\cdot 5\cdot 4}{1\cdot 2\cdot 3} = 20$

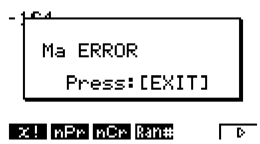
PROBability **nCr**

b) $\quad \dbinom{8}{4} = \dfrac{8\cdot 7\cdot 6\cdot 5}{1\cdot 2\cdot 3\cdot 4} = 70$

c) $\quad \dbinom{11}{7} = \dfrac{11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6\cdot 5}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7} = \dfrac{11\cdot 10\cdot 9\cdot 8}{1\cdot 2\cdot 3\cdot 4} = 330$

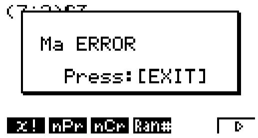
d) $\quad \dbinom{20}{19} = \dfrac{20\cdot 19\cdot \ldots \cdot 3\cdot 2}{1\cdot 2\cdot \ldots \cdot 18\cdot 19} = 20$

e) $\quad \dbinom{-1}{4} = \dfrac{-1\cdot(-2)\cdot(-3)\cdot(-4)}{1\cdot 2\cdot 3\cdot 4} = 1$



f) $\quad \dbinom{-2}{5} = \dfrac{-2\cdot(-3)\cdot(-4)\cdot(-5)\cdot(-6)}{1\cdot 2\cdot 3\cdot 4\cdot 5} = -6$

g) $\binom{7/2}{3}=\frac{(7/2)(5/2)(3/2)}{1\cdot 2\cdot 3}=\frac{35}{16}$



h) $\binom{1/2}{5}=\frac{(1/2)(-1/2)(-3/2)(-5/2)(-7/2)}{1\cdot 2\cdot 3\cdot 4\cdot 5}=\frac{7}{256}$

i) $\binom{-1/2}{4}=\frac{(-1/2)(-3/2)(-5/2)(-7/2)}{1\cdot 2\cdot 3\cdot 4}=\frac{35}{128}$

j) $\binom{-3/2}{6}=\frac{(-3/2)(-5/2)(-7/2)(-9/2)(-11/2)(-13/2)}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6}=\frac{3003}{1024}$

13.3-9.abcd

a)

Geometrisk rekke, $a_1=1$, $k=-x^3$, $\text{sum}=a_1\frac{1}{1-k}=1\,\frac{1}{1-(-x^3)}=\frac{1}{1+x^3}$

b)

Binomialrekke, $(1+x)^{\alpha}=\sum_{n=0}^{\infty}\binom{\alpha}{n}x^n$

$$\frac{1}{(1+x)^3}=(1+x)^{-3}=1+\binom{-3}{1}x+\binom{-3}{2}x^2+\binom{-3}{3}x^3+\ldots$$
$$=1-3x+6x^2-10x^3+\ldots$$

c)

Binomialrekke, $(1+x)^{\alpha}=\sum_{n=0}^{\infty}\binom{\alpha}{n}x^n$

$$\frac{1}{(3-x)^2}=\frac{1}{\left(3(1-\frac{x}{3})\right)^2}=\frac{1}{9}\left(1-\frac{x}{3}\right)^{-2}\qquad \text{bytter om } -\frac{x}{3}\rightarrow x$$
$$=\frac{1}{9}\left[1+\binom{-2}{1}\left(\frac{-x}{3}\right)+\binom{-2}{2}\left(\frac{-x}{3}\right)^2+\binom{-2}{3}\left(\frac{-x}{3}\right)^3+\ldots\right]$$
$$=\frac{1}{9}\left[1+\frac{2}{3}x+\frac{1}{3}x^2+\frac{4}{27}x^3+\ldots\right]$$
$$=\frac{1}{9}+\frac{2}{27}x+\frac{1}{27}x^2+\frac{4}{243}x^3+\ldots$$

d)

Binomialrekke, $(1+x)^{\alpha}=\sum_{n=0}^{\infty}\binom{\alpha}{n}x^n$

$$\sqrt[4]{1+x}=(1+x)^{\frac{1}{4}}=1+\binom{1/4}{1}x+\binom{1/4}{2}x^2+\binom{1/4}{3}x^3+\ldots$$
$$=1+\frac{1}{4}x-\frac{3}{32}x^2+\frac{7}{128}x^3-\ldots$$

13.3-10.abcd

a)

Binomialrekke, $(1+x)^{\alpha}=\sum_{n=0}^{\infty}\binom{\alpha}{n}x^n$

$$\frac{1}{(1+x)^2}=\binom{-2}{0}x^0+\binom{-2}{1}x^1+\binom{-2}{2}x^2+\binom{-2}{3}x^3+\ldots=1-2x+3x^2-4x^3+\ldots=\sum_{n=0}^{\infty}(-1)^n(n+1)x^n$$

b)

Binomialrekke, $(1+x)^{\alpha}=\sum_{n=0}^{\infty}\binom{\alpha}{n}x^n$

$$\frac{1}{(1+x)^4}=\binom{-4}{0}x^0+\binom{-4}{1}x^1+\binom{-4}{2}x^2+\binom{-4}{3}x^3+\binom{-4}{4}x^4+\binom{-4}{5}x^5+\ldots$$
$$=1+\frac{-4}{1}x+\frac{-4\cdot-5}{1\cdot2}x^2+\frac{-4\cdot-5\cdot-6}{1\cdot2\cdot3}x^3+\frac{-4\cdot-5\cdot-6\cdot-7}{1\cdot2\cdot3\cdot4}x^4+\frac{-4\cdot-5\cdot-6\cdot-7\cdot-8}{1\cdot2\cdot3\cdot4\cdot5}x^5\ldots$$
$$=\frac{3\cdot2\cdot1}{1\cdot2\cdot3}-\frac{4\cdot3\cdot2}{1\cdot2\cdot3}x+\frac{5\cdot4\cdot3}{1\cdot2\cdot3}x^2-\frac{6\cdot5\cdot4}{1\cdot2\cdot3}x^3+\frac{7\cdot6\cdot5}{1\cdot2\cdot3}x^4-\frac{8\cdot7\cdot6}{1\cdot2\cdot3}x^5\ldots$$
$$=\sum_{n=0}^{\infty}(-1)^n\binom{n+3}{3}x^n$$

c)

Binomialrekke, $(1+x)^{\alpha}=\sum_{n=0}^{\infty}\binom{\alpha}{n}x^n$

$$\sqrt[3]{1-x}=(1-x)^{\frac{1}{3}}=\binom{1/3}{0}(-x)^0+\binom{1/3}{1}(-x)^1+\binom{1/3}{2}(-x)^2+\binom{1/3}{3}(-x)^3+\binom{1/3}{4}(-x)^4+\ldots$$
$$=1-\frac{1}{3}x+\frac{1\cdot(-2)}{3\cdot3\cdot1\cdot2}x^2-\frac{1\cdot(-2)\cdot(-5)}{3\cdot3\cdot3\cdot1\cdot2\cdot3}x^3+\frac{1\cdot(-2)\cdot(-5)\cdot(-8)}{3\cdot3\cdot3\cdot3\cdot1\cdot2\cdot3\cdot4}x^4\ldots$$
$$=1-\frac{1}{3}x-\frac{1\cdot2}{3\cdot3\cdot1\cdot2}x^2-\frac{1\cdot2\cdot5}{3\cdot3\cdot3\cdot1\cdot2\cdot3}x^3-\frac{1\cdot2\cdot5\cdot8}{3\cdot3\cdot3\cdot3\cdot1\cdot2\cdot3\cdot4}x^4\ldots$$
$$=1-\frac{1}{3}x-\sum_{n=2}^{\infty}\frac{2\cdot5\cdot\ldots\cdot(3n-4)}{3^n\cdot n!}x^n$$

d)

Binomialrekke, $(1+x)^{\alpha}=\sum_{n=0}^{\infty}\binom{\alpha}{n}x^n$

$$\sqrt[4]{1+x}=(1+x)^{\frac{1}{4}}=\binom{1/4}{0}x^0+\binom{1/4}{1}(-x)^1+\binom{1/4}{2}x^2+\binom{1/4}{3}x^3+\binom{1/4}{4}x^4+\ldots$$
$$=1+\frac{1}{4}x+\frac{1\cdot(-3)}{4\cdot4\cdot1\cdot2}x^2+\frac{1\cdot(-3)\cdot(-7)}{4\cdot4\cdot4\cdot1\cdot2\cdot3}x^3+\frac{1\cdot(-3)\cdot(-7)\cdot(-11)}{4\cdot4\cdot4\cdot4\cdot1\cdot2\cdot3\cdot4}x^4\ldots$$
$$=1+\frac{1}{4}x-\frac{1\cdot3}{4\cdot4\cdot1\cdot2}x^2+\frac{1\cdot3\cdot7}{4\cdot4\cdot4\cdot1\cdot2\cdot3}x^3-\frac{1\cdot3\cdot7\cdot11}{4\cdot4\cdot4\cdot4\cdot1\cdot2\cdot3\cdot4}x^4\ldots$$
$$=1+\frac{1}{4}x+\sum_{n=2}^{\infty}(-1)^{n-1}\frac{3\cdot7\cdot\ldots\cdot(4n-5)}{3^n\cdot n!}x^n$$

13.3-11.abc

a)

$$f(x)=\ln(1+x)=x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+\ldots=\sum_{n=0}^{\infty}\frac{(-1)^{n+1}}{n}x^n \qquad \text{(Kjent rekke)}$$

$$f'(x)=\frac{1}{1+x}$$

$$g(x)=f'(x)=\frac{d}{dx}x+\frac{d}{dx}(\frac{-1}{2}x^2)+\frac{d}{dx}(\frac{1}{3}x^3)+\frac{d}{dx}(\frac{-1}{4}x^4)+\ldots$$
$$=1-x+x^2-x^3+\ldots=\sum_{n=0}^{\infty}(-1)^n x^n$$

b)

$$f(x)=\frac{1}{1-x}=1+x+x^2+x^3+\ldots+x^n+\ldots=\sum_{n=0}^{\infty}x^n$$ (Kjent rekke)

$$f'(x)=\frac{1}{(1-x)^2}$$

$$g(x)=f'(x)=0+1+2x+3x^2+4x^3+\ldots+(n+1)x^n+\ldots=\sum_{n=0}^{\infty}(1+n)x^n$$

c)

$$f(x)=\sqrt{1+x}=1+\frac{1}{2}x-\frac{1}{2^2 2!}x^2+\frac{1}{2^3 3!}x^3-\frac{1}{2^4 4!}x^4+\ldots$$ Se eksempel i læreboka
$$=1+\frac{1}{2}x+\sum_{n=2}^{\infty}(-1)^{n-1}\frac{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-3)}{2^n\cdot n!}x^n$$

$$f'(x)=\frac{1}{2\sqrt{1+x}}$$

$$g(x)=2\cdot f'(x)=2(0+\frac{1}{2}+n\sum_{n=2}^{\infty}(-1)^{n-1}\frac{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-3)}{2^n\cdot n!}x^{n-1})$$
$$=1+2n\sum_{n=2}^{\infty}(-1)^{n-1}\frac{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-3)}{2^n\cdot n!}x^{n-1}$$
$$=1+\sum_{n=2}^{\infty}(-1)^{n-1}\frac{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-3)}{2^{n-1}\cdot n-1!}x^{n-1}$$
$$=1+\sum_{n=1}^{\infty}(-1)^{n}\frac{1\cdot 3\cdot 5\cdot\ldots\cdot(2n-1)}{2^{n}\cdot n!}x^{n} \qquad (n\rightarrow n-1)$$

13.3-12.ab

a)

$$\cos x=1-\frac{1}{2!}x^2+\frac{1}{4!}x^4-\frac{1}{6!}x^6+\ldots=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n}$$ $$g(x)=\sin(3x)$$

$$f(x)=\cos(3x)=1-\frac{1}{2!}(3x)^2+\frac{1}{4!}(3x)^4-\frac{1}{6!}(3x)^6+\ldots=\sum_{n=0}^{\infty}\frac{(-1)^n 3^{2n}}{(2n)!}x^{2n}$$

$$\int_0^x f(t)\,dt=\int_0^x\cos(3t)\,dt=\frac{1}{3}\sin(3x)$$ (Bestemt integral fra 0 til x)

$$g(x)=3\int_0^x f(t)dt=3\int_0^x \cos(3t)dt$$
$$=3(\int_0^x 1\,dt+\int_0^x \frac{-1}{2!}3^2t^2dt+\int_0^x \frac{1}{4!}3^4t^4dt+\int_0^x \frac{-1}{6!}3^6t^6dt+...)$$
$$=3(x-\frac{1}{3!}3^2x^3+\frac{1}{5!}3^4x^5-\frac{1}{7!}3^6x^7+...)$$
$$=\frac{1}{1!}3^1x^1-\frac{1}{3!}3^3x^3+\frac{1}{5!}3^5x^5-\frac{1}{7!}3^7x^{7...}=\sum_{n=0}^{\infty}\frac{(-1)^n3^{2n+1}}{(2n+1)!}x^{2n+1}$$

b)

$$f(x)=\frac{1}{1-x^2}=1+x^2+x^4+x^6+...+x^{2n}+...=\sum_{n=0}^{\infty}x^{2n} \qquad g(x)=\ln\frac{1+x}{1-x}$$

$$\int_0^x \frac{1}{1-t^2}dt=\frac{1}{2}\ln(1+x)-\frac{1}{2}\ln(1-x)=\frac{1}{2}\ln(\frac{1+x}{1-x}) \qquad \text{(Bestemt integral fra 0 til } x\text{)}$$

$$g(x)=2\int_0^x \frac{1}{1-t^2}dt=2\int_0^x f(t)dt$$
$$=2(\int_0^x 1\,dt+\int_0^x t^2dt+\int_0^x t^4dt+\int_0^x t^6dt+...)$$
$$=2(x+\frac{1}{3}x^3+\frac{1}{5}x^5+\frac{1}{7}x^7...)$$
$$=\sum_{n=0}^{\infty}\frac{2}{2n+1}x^{2n+1}$$

13.4-1.abcdefg

a)

Følgefunksjon, $f(x)=\frac{x}{x^2+1}$

Integraltest: $\int_1^{\infty}\frac{x}{x^2+1}dx=\lim_{s\to\infty}\int_1^{s}\frac{x}{x^2+1}dx=\lim_{s\to\infty}\frac{1}{2}[\ln(x^2+1)]_1^s=\text{Eksisterer ikke}$, divergent.

b)

Følgefunksjon, $f(x)=\frac{x+1}{x^2}=\frac{1}{x}+\frac{1}{x^2}$

Integraltest: $\int_1^{\infty}\frac{x+1}{x^2}dx=\lim_{s\to\infty}\int_1^{s}\frac{x+1}{x^2}dx=\lim_{s\to\infty}[\ln(x)-\frac{1}{x}]_1^s=\text{Eksisterer ikke}$, divergens.

c)

Følgefunksjon, $f(x)=\frac{1}{x^2+1}$

Integraltest: $\int_1^{\infty}\frac{1}{x^2+1}dx=\lim_{s\to\infty}\int_1^{s}\frac{1}{x^2+1}dx=\lim_{s\to\infty}[\tan^{-1}x]_1^s=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$

d)

Følgefunksjon, $f(x)=\frac{1}{\sqrt{x+1}}$

Integraltest: $\int_1^{\infty} \frac{1}{\sqrt{x+1}} dx=\lim_{s \to \infty} \int_1^{s} \frac{1}{\sqrt{x+1}} dx=\lim_{s \to \infty} 2[\sqrt{x+1}]_1^{\infty}=\text{Eksisterer ikke}$, divergens.

e)

Følgefunksjon, $f(x)=\frac{1}{(x+1)^3}$

Integraltest: $\int_1^{\infty} \frac{1}{(x+1)^3} dx=\lim_{s \to \infty} \int_1^{s} \frac{1}{(x+1)^3} dx=\lim_{s \to \infty} -\frac{1}{2}[\frac{1}{(x+1)^2}]_1^s=0-(-\frac{1}{8})=\frac{1}{8}$, konvergens.

f)

Følgefunksjon, $f(x)=\frac{1}{x(x+1)}$

Integraltest: $\int_1^{\infty} \frac{1}{x(x+1)} dx=\lim_{s \to \infty} \int_1^{s} \frac{1}{x(x+1)} dx=\lim_{s \to \infty} [\ln x-\ln(x+1)]_1^s$

$=0-(0-\ln 2)=\ln 2 \qquad \text{Konvergens}$

g)

Følgefunksjon, $f(x)=\frac{x}{e^{x^2}}$

Integraltest: $\int_1^{\infty} \frac{x}{e^{x^2}} dx=\lim_{s \to \infty} \int_1^{s} \frac{x}{e^{x^2}} dx=\lim_{s \to \infty} -\frac{1}{2}[\frac{1}{e^{x^2}}]_1^s=-\frac{1}{2}[0-\frac{1}{e}]=\frac{1}{2e} \qquad \text{Konvergens}$

13.4-2.abcdefg

a)

Leddene $a_n=\frac{1}{n^2+2n+1}$ sammenliknes med leddene i *p*-rekka $b_n=\frac{1}{n^2}$ som konvergerer.

Vi ser at $a_n < b_n$, og vi har konvergens.

b)

Leddene $a_n=\frac{n}{(n+1)(n+2)}$ sammenliknes med leddene i *p*-rekka med ledd $b_n=\frac{1}{n^1}$, som også er kjent som den harmoniske rekka som divergerer.

Omformer $a_n=\frac{n}{(n+1)(n+2)}=\frac{n}{n^2+3n+2}=\frac{1}{n+3+\frac{2}{n}} \xrightarrow[\text{store } n]{} \frac{1}{n}$

Vi ser at $a_n \approx b_n$ for store *n*-verdier og vi har divergens.

c)

Leddene $a_n=\frac{5n^2}{n^3+n+3}$ sammenliknes med leddene i *p*-rekka med ledd $b_n=\frac{1}{n^1}$, som også er kjent som den harmoniske rekka som divergerer.

Omformer $a_n = \frac{5n^2}{n^3+n+3} = 5\frac{1}{n+\frac{1}{n}+\frac{3}{n^2}} \underset{\text{store } n}{\rightarrow} 5\frac{1}{n}$

Vi ser at $a_n \approx 5 b_n$ for store *n*-verdier og vi har divergens.

d)

Leddene $a_n = \frac{\sqrt{n}}{n^2+2}$ sammenliknes med leddene i *p*-rekka med ledd $b_n = \frac{1}{n^p}$.

Omformer $a_n = \frac{\sqrt{n}}{n^2+2} = \frac{1}{n^{\frac{3}{2}}+\frac{2}{\sqrt{n}}} \underset{\text{store } n}{\rightarrow} \frac{1}{n^{\frac{3}{2}}}$

Vi ser at $a_n \approx \frac{1}{n^p}$ for store *n*-verdier og vi har konvergens fordi $p>1$.

e)

Leddene $a_n = \frac{4}{\sqrt{n+7}}$ sammenliknes med leddene i *p*-rekka med ledd $b_n = \frac{1}{n^p}$

Omformer $a_n = \frac{4}{\sqrt{n+7}} = 4\frac{1}{(n+7)^{\frac{1}{2}}} \underset{\text{store } n}{\rightarrow} 4\frac{1}{n^{\frac{1}{2}}}$

Vi ser at $a_n \approx 4\frac{1}{n^p}$ for store *n*-verdier og vi har divergens fordi $p<1$.

f)

Leddene $a_n = \frac{\sin^2 n}{n^2+5n}$ sammenliknes med leddene i *p*-rekka med ledd $b_n = \frac{1}{n^p}$

Omformer $a_n = \frac{\sin^2 n}{n^2+5n} \underset{\text{store } n}{\rightarrow} \frac{\sin^2 n}{n^2} \leq \frac{1}{n^2}$

Vi ser at $a_n \leq \frac{1}{n^p}$ for store *n*-verdier og vi har konvergens fordi $p>1$.

g)

Leddene $a_n = \frac{1}{1+5^n}$ sammenliknes med leddene i rekka med ledd $b_n = \frac{1}{5^n}$ som er en geometrisk rekke med $k = \frac{1/5^{n+1}}{1/5^n} = \frac{1}{5} < 1$ som konvergerer.

Vi ser at $a_n < b_n$ for alle *n*-verdier og vi har konvergens fordi $p>1$.

13.4-3.abcdefg

a)

$a_n = \frac{3}{5^n}$, $a_{n+1} = \frac{3}{5^{n+1}}$

Tester, $k = \lim_{n\rightarrow\infty} \frac{a_{n+1}}{a_n} = \lim_{n\rightarrow\infty} \left(\frac{3}{5^{n+1}} \cdot \frac{5^n}{3}\right) = \lim_{n\rightarrow\infty} \left(\frac{3}{5^{n+1}} \cdot \frac{5^n}{3}\right) = \frac{1}{5}$ $k < 1$, vi har konvergens.

b)

$$a_n=\frac{2^n}{n} \quad , \quad a_{n+1}=\frac{2^{n+1}}{n+1}$$

Tester, $k=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\left(\frac{2^{n+1}}{n+1}\cdot\frac{n}{2^n}\right)=2\cdot\lim_{n\to\infty}\frac{n}{n+1}=2\cdot1=2$ $k>1$, vi har divergens.

c)

$$a_n=\frac{4n+1}{3^n} \quad , \quad a_{n+1}=\frac{4(n+1)+1}{3^{n+1}}$$

Tester, $k=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\left(\frac{4n+5}{3^{n+1}}\cdot\frac{3^n}{4n+1}\right)=\frac{1}{3}\cdot\lim_{n\to\infty}\frac{4n+1}{4n+5}=\frac{1}{3}\cdot1=\frac{1}{3}$ $k<1$, konvergens.

d)

$$a_n=\frac{n^2+3}{2^n} \quad , \quad a_{n+1}=\frac{(n+1)^2+3}{2^{n+1}}$$

Tester, $k=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\left(\frac{(n+1)^2+3}{2^{(n+1)}}\cdot\frac{2^n}{n^2+3}\right)=\frac{1}{2}\cdot\lim_{n\to\infty}\frac{n^2+2n+4}{n^2+3}=\frac{1}{2}$ $k<1$, konvergens.

e)

$$a_n=\frac{7^n}{n^n} \quad , \quad a_{n+1}=\frac{7^{n+1}}{(n+1)^{n+1}}$$

Tester, $k=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{7^{n+1}}{(n+1)^{n+1}}\cdot\frac{n^n}{7^n}=7\cdot\lim_{n\to\infty}\frac{n^n}{(n+1)^n\cdot(n+1)}$ $k<1$, konvergens.

$$=7\cdot\lim_{n\to\infty}\left(\frac{n}{n+1}\right)^n\cdot\frac{1}{n+1}=7\cdot1\cdot0=0$$

f)

$$a_n=\frac{(\ln 5)^n}{n} \quad , \quad a_{n+1}=\frac{(\ln 5)^{n+1}}{n+1}$$

Tester, $k=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{(\ln 5)^{n+1}}{n+1}\cdot\frac{n}{\ln 5}=\ln 5\cdot\lim_{n\to\infty}\frac{n}{n+1}=(\ln 5)\cdot1=\ln 5\approx1.6$ $k>1$, divergens.

g)

$$a_n=\frac{5^n}{n!} \quad , \quad a_{n+1}=\frac{5^{n+1}}{(n+1)!}$$

Tester, $k=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{5^{n+1}}{(n+1)!}\cdot\frac{n!}{5^n}=5\cdot\lim_{n\to\infty}\frac{1}{n+1}=5\cdot0=0$ $k<1$, konvergens.

13.4-4.abcdefg

a)

Forholdstesten,

$$a_n=\frac{n^2}{e^n} \quad , \quad a_{n+1}=\frac{(n+1)^n}{e^{n+1}}$$

Tester, $k=\lim\limits_{n\to\infty}\dfrac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\dfrac{(n+1)^2}{e^{n+1}}\cdot\dfrac{e^n}{n^2}=\dfrac{1}{e}\cdot\lim\limits_{n\to\infty}\dfrac{n^2+2n+1}{n^2}=\dfrac{1}{e}$ $k < 1$, konvergens.

b)

Leddene $a_n=\dfrac{2n+3}{n^2+3n+1}$ sammenliknes med leddene i *p*-rekka med ledd $b_n=\dfrac{1}{n^1}$, som også er kjent som den harmoniske rekka som divergerer.

Omformer $a_n=\dfrac{2n+3}{n^2+3n+1}=2\dfrac{1+\dfrac{3}{n}}{n+3+\dfrac{1}{n}}\underset{\text{store } n}{\to} 2\dfrac{1}{n}$

Vi ser at $a_n\approx 2b_n$ for store *n*-verdier og vi har divergens.

c)

Leddene $a_n=\dfrac{1}{n+n^{5/2}}$ sammenliknes med leddene i *p*-rekka med ledd $b_n=\dfrac{1}{n^p}$, som konvergerer hvis $p > 1$.

Omformer $a_n=\dfrac{1}{n+n^{5/2}}\underset{\text{store } n}{\to}\dfrac{1}{n^{5/2}}$

Vi ser at $a_n\approx b_n$ med $p > 1$ for store *n*-verdier og vi har konvergens.

d)

Forholdstesten,

$a_n=\dfrac{n!}{6^n}$, $a_{n+1}=\dfrac{(n+1)!}{6^{n+1}}$

Tester, $k=\lim\limits_{n\to\infty}\dfrac{a_{n+1}}{a_n}=\lim\limits_{n\to\infty}\dfrac{(n+1)!}{6^{n+1}}\cdot\dfrac{6^n}{n!}=\dfrac{1}{6}\cdot\lim\limits_{n\to\infty}(n+1)=\text{Over alle grenser}$ $k > 1$, divergens.

e)

Om vi prøver med integraltesten kommer vi til et integral som ikke kan løses, iallefall med vanlig kalkulus. Forholdstesten går bra, men gir grenseverdi lik 1, altså ingen konklusjon. Vi må bruke sammenlikningstesten.

Leddene $a_n=\dfrac{1}{5+\ln n}$ sammenliknes med leddene i *p*-rekka med ledd $b_n=\dfrac{1}{n^1}$, som også er kjent som den harmoniske rekka som divergerer.

Vi kan vise at $0<\ln n<n,$, $n=1,2,3,\ldots$ for eksempel ved å drøfte $\lim\limits_{n\to\infty}\dfrac{\ln n}{n}$, og om nevneren i a_n bare var $\ln n$ ville løsningen være grei. Sammenlikningstesten slik den er formulert i læreboka gjelder 'for alle *n*'. Det e ikke de første leddene i ei rekke som avgjør om den konvergerer eller divergerer, det er hvordan utviklingen er utover i rekka. Sammenlikningstesten kan formuleres som i læreboka, men drøftes for 'alle *n* større enn et passe heltall *N*'. om vi se på verdien av nevnerne i a_n og b_n har vi

n	1	2	3	4	5	6	7	8	9	10
$5+\ln n$	5	5,69	6,098	6,386	6,609	6,79	6,94	7,079	7,197	7,30

Konklusjon, $\frac{1}{5+\ln n} > \frac{1}{n^1}$ for $n>7$ – rekka divergerer.

f)

Leddene $a_n = \frac{3+\sin n}{n^2}$ sammenliknes med leddene i *p*-rekka med ledd $b_n = \frac{1}{n^2}$, som konvergerer.

Vi har at $-1 \le \sin n \le 1$, som gir at $a_n \le \frac{4}{n^2}$ og $a_n \le K \cdot b_n$, og dermed konvergens.

g)

Slår til med en integraltest(!) med $f(x) = \frac{1}{x\sqrt{\ln x}}$,

$$\int_1^\infty \frac{1}{x\sqrt{\ln x}}dx = \lim_{s\to\infty}\int_1^s \frac{1}{x\sqrt{\ln x}}dx$$

$$((u = \ln x \qquad du = \frac{1}{x}dx \qquad dx = x\,du))$$

$$= \lim_{s\to\infty}\int_1^s u^{-\frac{1}{2}}du = \lim_{s\to\infty}[2u^{\frac{1}{2}}]_{x=1}^{x=s} = \lim_{s\to\infty}[2\sqrt{\ln x}]_1^s = \text{Over alle grenser, divergens.}$$

13.4-5.abcdefg

a)

En *alternerende* rekke med ledd a_n konvergerer hvis

1 - $|a_n| \ge |a_{n+1}|$

2 - $\lim_{n\to\infty} a_n = 0$

Tester 1: $\frac{1}{n+2} > \frac{1}{n(n+1)+2}$, OK

Tester 2: $\lim_{n\to\infty}\frac{1}{n+2} = 0$, OK - konvergens.

b)

En *alternerende* rekke med ledd a_n konvergerer hvis

1 - $|a_n| \ge |a_{n+1}|$

2 - $\lim_{n\to\infty} a_n = 0$

Tester 1: $\frac{1}{n^3+1} > \frac{1}{(n+1)^3+1}$, OK

Tester 2: $\lim_{n\to\infty}\frac{1}{n^3+1} = 0$, OK - konvergens.

c)

En *alternerende* rekke med ledd a_n konvergerer hvis

1 - $|a_n| \ge |a_{n+1}|$

2 - $\lim_{n\to\infty} a_n = 0$

Tester 1: $a_n - a_{n+1} = \frac{n}{3^n} - \frac{n+1}{3^{n+1}} = \frac{3n-(n+1)}{3^{n+1}} = \frac{2n-1}{3^{n+1}} = POS$, OK

Tester 2: $\lim_{n\to\infty} \frac{n}{3^n} = \frac{'\infty'}{'\infty'}$ Drøfter $f(x) = \frac{x}{3^x}$,

$\lim_{x\to\infty} \frac{x}{3^x} = \frac{'\infty'}{'\infty'} = \text{l'Hopital} = \lim_{x\to\infty} \frac{1}{3^x \cdot \ln 3} = 0$, konvergens.

d)

En *alternerende* rekke med ledd a_n konvergerer hvis

1 - $|a_n| \geq |a_{n+1}|$

2 - $\lim_{n\to\infty} a_n = 0$

Har en mistanke om at test 2 svikter, drøfter $f(x) = \frac{x}{\sqrt{x^2+3}}$,

$\lim_{x\to\infty} \frac{x}{\sqrt{x^2+3}} = \lim_{x\to\infty} \frac{\sqrt{x^2}}{\sqrt{x^2+3}} = \lim_{x\to\infty} \sqrt{\frac{x^2}{x^2+3}} = 1$, $\neq 0$, divergens.

e)

En *alternerende* rekke med ledd a_n konvergerer hvis

1 - $|a_n| \geq |a_{n+1}|$

2 - $\lim_{n\to\infty} a_n = 0$

Tester 1: Ser på $\frac{a_n}{a_{n+1}} = \frac{n\sqrt{3^{n+1}+1}}{(n+1)\sqrt{3^n+1}} = \frac{\sqrt{3n^2 3^n + n^2}}{\sqrt{(n+1)^2 3^n + (n+1)^2}} > 1$, $a_n > a_{n+1}$, OK

(Testen svikter for *n*=1, men gjelder for alle *n*>1, og det holder.)

Tester 2: $\lim_{n\to\infty} \frac{n}{\sqrt{3^n+1}} = \lim_{n\to\infty} \frac{n}{\sqrt{3^n}} = \lim_{n\to\infty} \frac{n}{3^{\frac{n}{2}}} = 0$, konvergens.

f)

En *alternerende* rekke med ledd a_n konvergerer hvis

1 - $|a_n| \geq |a_{n+1}|$

2 - $\lim_{n\to\infty} a_n = 0$

Tester 1: Nevneren er alltid positiv, men telleren har verdier $-1 \leq \cos n \leq 1$, og gir både positive og negative verdier. Test 1 svikter, rekka divergerer (hvis definisjonen tolkes strengt).

g)

En *alternerende* rekke med ledd a_n konvergerer hvis

1 - $|a_n| \geq |a_{n+1}|$

2 - $\lim_{n\to\infty} a_n = 0$

Har en mistanke om at test 2 svikter, drøfter $f(x)=\frac{x}{\ln x}$,

$$\lim_{x\to\infty}\frac{x}{\ln x}=\frac{'\infty'}{'\infty'}=\text{l'Hoptial}=\lim_{x\to\infty}\frac{1}{\frac{1}{x}}=\lim_{x\to\infty}x=\text{Over alle grenser}$$, divergens.

13.4-6.abcd

a)

En rekke med ledd a_n konvergerer *absolutt* dersom $\sum_{n=1}^{\infty}|a_n|$ konvergerer.

En *alternerende* rekke med ledd $(-1)^n a_n$ konvergerer *betinget* dersom $\sum_{n=1}^{\infty}(-1)^n a_n$ konvergerer, men $\sum_{n=1}^{\infty}|(-1)^n|a_n$ divergerer.

Rekka $\sum_{n=1}^{\infty}(-1)^n\frac{1}{4n^2+3}$ <u>konvergerer absolutt</u> fordi leddene avtar raskere enn $\frac{1}{4n^2}$ om vi sammeligner med *p*-rekka med *p* = 2.

b)

Rekka $\sum_{n=1}^{\infty}(-1)^n\frac{1}{5n-1}$ <u>konvergerer betinget</u> fordi leddene avtar som $\frac{1}{4n}$ om vi sammeligner med *p*-rekka med *p* = 1.

c)

Rekka $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{\cos^2 n}{3^n}$ <u>konvergerer absolutt</u> fordi leddenes absoluttverdi avtar raskere enn $\frac{1}{3^n}$, en geometrisk rekke med $k=\frac{1}{3}$.

d)

Rekka $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{\sin^2 n+1}$ <u>divergerer</u> fordi leddenes verdi ikke avtar utover i rekka, men har verdier spredt mellom 1 og 1/2.

13.5-1.abcd

a)

Dette er ei geometrisk rekke med $k=x$ som konvergerer for $|x|<R=1$.

b)

Undersøker konvergens med forholdstesten,

$$k=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{((n+1)^2+3)x^{(n+1)}}{(n^2+3)x^n}=\lim_{n\to\infty}x\frac{n^2+2n+4}{n^2+3}=\lim_{n\to\infty}x\frac{1+\frac{2}{n}+\frac{4}{n^2}}{1+\frac{3}{n^2}}=x$$

Konvergens for $|x|<R=1$.

c)

Undersøker konvergens med forholdstesten,

$$k=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{x^{n+1}}{n+1+4}\cdot\frac{n+4}{x^n}=\lim_{n\to\infty}x\frac{n+4}{n+5}=x$$

Konvergens for $|x|<R=1$.

d)

Undersøker konvergens med forholdstesten,

$$k=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{x^{n+1}}{(n+1)^2+1}\cdot\frac{n^2+1}{x^n}=\lim_{n\to\infty}x\frac{n^2+1}{n^2+2n+2}=x$$

Konvergens for $|x|<R=1$.

13.5-2.abcde

a)

Undersøker konvergens med forholdstesten,

$$k=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{2^{n+1+1}\cdot x^{n+1}}{(n+1)\cdot 5^{n+1}}\cdot\frac{n\cdot 5^n}{2^{n+1}\cdot x^n}=\lim_{n\to\infty}\frac{2}{5}x\frac{n}{n+1}=\frac{2}{5}x$$

Konvergens for $\frac{2}{5}|x|<1$, $-\frac{5}{2}<x<\frac{5}{2}$.

b)

Undersøker konvergens med forholdstesten,

$$k=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{(n+1+2)\cdot x^{n+1}}{((n+1)^3+1)\cdot 4^{n+1}}\cdot\frac{(n^3+1)\cdot 4^n}{(n+2)\cdot x^n}=\lim_{n\to\infty}\frac{1}{4}x\frac{n^4+3n^3+n+3}{n^4+5n^3+9n^2+8n+4}=\frac{1}{4}x$$

Konvergens for $\frac{1}{4}|x|<1$, $-4<x<4$.

c)

Undersøker konvergens med forholdstesten,

$$k=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{(n+1)(x-1)^{n+1}}{3^{n+1}}\cdot\frac{3^n}{n(x-1)^n}=\lim_{n\to\infty}\frac{1}{3}(x-1)\frac{n+1}{n}=\frac{1}{3}(x-1)$$

Konvergens for $\frac{1}{3}|x-1|<1$, $-1<\frac{1}{3}(x-1)<1$, $-2<x<4$.

d)

Undersøker konvergens med forholdstesten,

$$k=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=-\lim_{n\to\infty}\frac{(x-1)^{n+1}}{(n+1)^2}\cdot\frac{n^2}{(x+1)^n}=-\lim_{n\to\infty}(x+1)\frac{n^2}{(n+1)^2}=-x-1$$

Konvergens for $|-x-1|<1$, $-1<-x-1<1$, $-2<x<0$.

e)

Undersøker konvergens med forholdstesten,

$$k=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{2^{n+1}(x-2)^{n+1}}{(n+1)!}\cdot\frac{n!}{2^n(x-2)^n}=\lim_{n\to\infty}2(x-2)\frac{1}{n+1}=0$$

Konvergens for alle verdier av *x*.

13.5-3.abcde

a)

Undersøker konvergens med forholdstesten,

$$k=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{x^{4(n+1)}}{16^{n+1}}\cdot\frac{16^n}{x^{4n}}=\lim_{n\to\infty}\frac{1}{16}x^4=\frac{1}{16}x^4$$

Konvergens for $\frac{1}{16}|x^4|<1$, $-2<x<2$ og evt. ± 2 .

Nedre grense, $a_n=\frac{(-2)^{4n}}{16^n}=1$, gir divergerende rekke

Øvre grense, $a_n=\frac{2^{4n}}{16^n}=1$, gir divergerende rekke

Konvergens for $-2<x<2$.

b)

Undersøker konvergens med forholdstesten,

$$k=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{x^{n+1}}{2^{n+1}\left((n+1)^2+1\right)}\cdot\frac{2^n\left(n^2+1\right)}{x^n}=\lim_{n\to\infty}\frac{1}{2}\cdot\frac{n^2+1}{n^2+2n+2}x=\frac{1}{2}x$$

Konvergens for $\frac{1}{2}|x|<1$, $-2<x<2$ og evt. ± 2 .

Nedre grense, $a_n=\frac{(-2)^n}{2^n\left(n^2+1\right)}=\frac{(-1)^n}{n^2+1}$, alternerende rekke, avtakende ledd

Øvre grense, $a_n=\frac{2^n}{2^n\left(n^2+1\right)}=\frac{1}{n^2+1}$, gir konvergens, sml. $a_n=\frac{1}{n^p}$

Konvergens for $-2\le x\le 2$.

c)

Undersøker konvergens med forholdstesten,

$$k=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{(n+1)x^{2(n+1)}}{9^{n+1}}\cdot\frac{9^n}{n x^{2n}}=\lim_{n\to\infty}\frac{1}{9}\cdot\frac{n+1}{n}x^2=\frac{1}{9}x^2$$

Konvergens for $\frac{1}{9}|x^2|<1$, $-3<x<3$ og evt. ± 3 .

Nedre grense, $a_n=\frac{n(-3)^{2n}}{9^n}=n$, gir divergerende rekke

Øvre grense, $a_n=\frac{n\,3^{2n}}{9^n}=n$, gir divergerende rekke

Konvergens for $-3<x<3$.

d)

Undersøker konvergens med forholdstesten,

$$k=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{(n+1)!\cdot x^{n+1}}{(n+1+2)!}\cdot\frac{(n+2)!}{n!\cdot x^n}=\lim_{n\to\infty}\frac{n+1}{n+3}x=x$$

Konvergens for $|x|<1$, $-1<x<1$ og evt. ± 1 .

Nedre grense, $a_n=-\frac{n!}{(n+2)!}=-\frac{1}{(n+1)(n+2)}=-\frac{1}{n^2+3n+2}$, danner konvergerende rekke, sml. $a_n=\frac{1}{n^p}$

Øvre grense, $a_n=\frac{n!}{(n+2)!}=\frac{1}{(n+1)(n+2)}=\frac{1}{n^2+3n+2}$, danner konvergerende rekke, sml. $a_n=\frac{1}{n^p}$

Konvergens for $-1\le x\le 1$.

e)

Undersøker konvergens med forholdstesten,

$$k=\lim_{n\to\infty}\frac{a_{n+1}}{a_n}=\lim_{n\to\infty}\frac{(x-1)^{n+1}}{2^{n+1}\cdot(n+1+1)}\cdot\frac{2^n(n+1)}{(x-1)^n}=\frac{1}{2}\lim_{n\to\infty}\frac{n+1}{n+2}(x-1)=\frac{1}{2}(x-1)$$

Konvergens for $\frac{1}{2}|x-1|<1$, $-1<x<3$ og evt. -1, 3.

Nedre grense, $a_n=\frac{(-2)^n}{2^n(n+1)}=\frac{(-1)^n}{n+1}$, danner konvergerende (alternerende) rekke, sml. $a_n=\frac{1}{n^p}$.

Øvre grense, $a_n=\frac{2^n}{2^n(n+1)}=\frac{1}{n+1}$ danner divergerende rekke, sml. $a_n=\frac{1}{n^p}$

Konvergens for $-1\le x<3$.

13.6-2.ab

a)

$$e^x\approx 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}$$

$$e^{-0.2}\approx 1+(-0.2)+\frac{(-0.2)^2}{2!}+\frac{(-0.2)^3}{3!}=0.8187$$

b)

$$\ln(1+x)\approx x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4$$

$$\ln(1.3)=\ln(1+0.3)\approx 0.3-\frac{1}{2}0.3^2+\frac{1}{3}0.3^3-\frac{1}{4}0.3^4=0.2620$$

13.6-3.ab

a)

$$\sin x \approx x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5$$

$$\sin\frac{18\cdot\pi}{180} = \sin 0.314159 \approx 0.314159 - \frac{1}{3!}0.314159^3 + \frac{1}{5!}0.314159^5 = 0.30902$$

b)

$$\cos x \approx 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4$$

$$\cos\frac{18\cdot\pi}{180} = \cos 0.314159 \approx 1 - \frac{1}{2!}0.314159^2 + \frac{1}{4!}0.314159^4 = 0.95106$$

13.6-4.abcd

a)

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \ldots = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+1} \quad \text{for alle } x$$

$$\frac{\sin t}{t} = \frac{t}{t} - \frac{1}{3!}\frac{t^3}{t} + \frac{1}{5!}\frac{t^5}{t} - \frac{1}{7!}\frac{t^7}{t} + \ldots = 1 - \frac{1}{3!}t^2 + \frac{1}{5!}t^4 - \frac{1}{7!}t^6 + \ldots$$

$$g(x) = \int_0^x \frac{\sin t}{t}\,dt = x - \frac{1}{3\cdot 3!}x^3 + \frac{1}{5\cdot 5!}x^5 - \frac{1}{7\cdot 7!}x^7 + \ldots = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)\cdot(2n+1)!}x^{2n+1}$$

b)

$$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \ldots = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n} \quad \text{for alle } x$$

$$1 - \cos t = 1 - (1 - \frac{1}{2!}t^2 + \frac{1}{4!}t^4 - \frac{1}{6!}t^6 + \ldots) = \frac{1}{2!}t^2 - \frac{1}{4!}t^4 + \frac{1}{6!}t^6 - \ldots$$

$$\frac{1-\cos t}{t} = \frac{1}{2!}t - \frac{1}{4!}t^3 + \frac{1}{6!}t^5 - \ldots$$

$$g(x) = \int_0^x \frac{1-\cos t}{t}\,dt = \frac{1}{2\cdot 2!}x^2 - \frac{1}{4\cdot 4!}x^4 + \frac{1}{6\cdot 6!}x^6 - \ldots = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n\cdot(2n)!}x^{2n}$$

c)

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$

$$e^t - 1 = t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \ldots \qquad \Rightarrow \qquad \frac{e^t - 1}{t} = 1 + \frac{t}{2!} + \frac{t^2}{3!} + \frac{t^3}{4!} + \ldots$$

$$g(x) = \int_0^x \frac{e^t - 1}{t}\,dt = x + \frac{x^2}{2\cdot 2!} + \frac{x^3}{3\cdot 3!} + \frac{x^4}{4\cdot 4!} = \sum_{n=1}^{\infty}\frac{1}{n\cdot(n)!}x^n$$

d)

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \ldots$$

$$\sin(t^2) = t^2 - \frac{1}{3!}(t^2)^3 + \frac{1}{5!}(t^2)^5 - \frac{1}{7!}(t^2)^7 + \ldots = t^2 - \frac{1}{3!}t^6 + \frac{1}{5!}t^{10} - \frac{1}{7!}t^{14} + \ldots$$

$$g(x)=\int_0^x \sin(t^2)\,dt=\frac{1}{3}x^3-\frac{1}{7\cdot 3!}x^7+\frac{1}{11\cdot 5!}x^{11}-\frac{1}{15\cdot 7!}x^{15}+\ldots=\sum_{n=0}^{\infty}\frac{(-1)^n}{(4n+3)\cdot(2n+1)!}\cdot x^{4n+3}$$

($2n+1$ for $n=0,1,2,3,\ldots$: $1,3,5,7,\ldots$)
($4n+3$ for $n=0,1,2,3,\ldots$: $3,7,11,15,\ldots$)

13.6-5.ab

a)

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots \qquad \Rightarrow \qquad e^{t^2}=1+t^2+\frac{t^4}{2!}+\frac{t^6}{3!}+\frac{t^8}{4!}+\ldots$$

$$t\,e^{t^2}=t+t\cdot t^2+t\frac{(t^2)^2}{2!}+t\frac{(t^2)^3}{3!}+t\frac{(t^2)^4}{4!}+\ldots=t+t^3+\frac{t^5}{2!}+\frac{t^7}{3!}+\frac{t^9}{4!}+\ldots$$

$$\int_0^1 t\,e^{t^2}\,dt=\left[\frac{1}{2}t^2+\frac{1}{4}t^4+\frac{1}{6}\frac{t^6}{2!}+\frac{1}{8}\frac{t^8}{3!}+\frac{1}{10}\frac{t^{10}}{4!}+\ldots\right]_0^1$$

$$=\frac{1}{2}+\frac{1}{4}+\frac{1}{6\cdot 2!}+\frac{1}{8\cdot 3!}+\frac{1}{10\cdot 4!}+\ldots=\frac{1}{2}+\frac{1}{2\cdot 2!}+\frac{1}{2\cdot 3!}+\frac{1}{2\cdot 4!}+\frac{1}{2\cdot 5!}+\ldots=\frac{1}{2}\sum_{n=1}^{\infty}\frac{1}{n!}$$

b)

$$\ln(1+x)=x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+\ldots$$

$$\ln(1-t)=-t-\frac{1}{2}(-t)^2+\frac{1}{3}(-t)^3-\frac{1}{4}(-t)^4+\ldots=-(t+\frac{1}{2}t^2+\frac{1}{3}t^3+\frac{1}{4}t^4+\ldots)$$

$$\ln(1-t^2)=-(t^2+\frac{1}{2}t^4+\frac{1}{3}t^6+\frac{1}{4}t^8+\ldots)$$

$$\int_0^{1/2}\ln(1-t^2)\,dt=\left[-(\frac{1}{3}t^3+\frac{1}{2}\frac{1}{5}t^5+\frac{1}{3}\frac{1}{7}t^7+\frac{1}{4}\frac{1}{9}t^9+\ldots)\right]_0^{1/2}$$

$$=-(\frac{1}{3}\frac{1}{2^3}+\frac{1}{2}\frac{1}{5}\frac{1}{2^5}+\frac{1}{3}\frac{1}{7}\frac{1}{2^7}+\frac{1}{4}\frac{1}{9}\frac{1}{2^9}+\ldots)=-\sum_{n=1}^{\infty}\frac{1}{n(2n+1)2^{2n+1}}$$

13.6-6.abcd

a)

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots=\sum_{n=0}^{\infty}\frac{x^n}{n!}$$

$$e^3=1+3+\frac{3^2}{2!}+\frac{3^3}{3!}+\frac{3^4}{4!}+\ldots=\sum_{n=0}^{\infty}\frac{3^n}{n!}$$

b)

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots=\sum_{n=0}^{\infty}\frac{x^n}{n!}$$

$$e^{-5}=1-5+\frac{(-5)^2}{2!}+\frac{(-5)^3}{3!}+\frac{(-5)^4}{4!}+\ldots=\sum_{n=0}^{\infty}(-1)^n\frac{5^n}{n!}$$

c)

$$\sin x=x-\frac{1}{3!}x^3+\frac{1}{5!}x^5-\frac{1}{7!}x^7+\ldots$$

$$\sin\left(\frac{1}{2}\right)=\frac{1}{2}-\frac{1}{3!}\frac{1}{2^3}+\frac{1}{5!}\frac{1}{2^5}-\frac{1}{7!}\frac{1}{2^7}+\ldots=\sum_{n=0}^{\infty}\frac{(-1)^n}{2^{2n+1}(2n+1)!}$$

d)

$$\cos x=1-\frac{1}{2!}x^2+\frac{1}{4!}x^4-\ldots$$

$$\cos\frac{\pi}{2}=0=1-\frac{\pi^2}{2^2 2!}+\frac{\pi^4}{2^4 4!}-\ldots=\sum_{n=0}^{\infty}\frac{(-1)^n\pi^{2n}}{2^{2n}(2n)!}$$

13.6-7.abc

a)

$$\ln(1+x)=x-\frac{1}{2}x^2+\frac{1}{3}x^3-\frac{1}{4}x^4+\ldots$$

$$x\ln(1+x)=x^2-\frac{1}{2}x^3+\frac{1}{3}x^4-\frac{1}{4}x^5+\ldots=\sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^{n+1}}{n}$$

b)

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ldots$$

$$e^{-x}=1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}-\ldots$$

$$x^2e^{-x}=x^2-x^3+\frac{x^4}{2!}-\frac{x^3}{3!}+\frac{x^6}{4!}+\ldots=\sum_{n=0}^{\infty}(-1)^n\frac{x^{n+2}}{n!}$$

c)

$$\sin x=x-\frac{1}{3!}x^3+\frac{1}{5!}x^5-\frac{1}{7!}x^7+\ldots$$

$$\sin x^2=x^2-\frac{1}{3!}x^6+\frac{1}{5!}x^{10}-\frac{1}{7!}x^{14}+\ldots$$

$$\frac{1}{x^2}\sin x^2=1-\frac{1}{3!}x^4+\frac{1}{5!}x^8-\frac{1}{7!}x^{12}+\ldots=\sum_{n=0}^{\infty}(-1)^n\frac{x^{4n}}{(2n+1)!}$$

14.3-1.abcdef

a)

$$F(s)=\int_0^4 3e^{-st}\,dt+\int_4^{\infty}0\cdot e^{-st}\,dt=3\left[-\frac{1}{s}e^{-st}\right]_0^4=\frac{3}{s}\left(1-e^{-4s}\right)$$

b)

$$F(s)=\int_0^2 0\,dt+\int_2^4 5e^{-st}\,dt+\int_5^{\infty}0\,dt=5\left[-\frac{1}{s}e^{-st}\right]_0^4=\frac{5}{s}\left(e^{-2s}-e^{-4s}\right)$$

c)

$$\int_0^3 4e^{-st}dt + \int_3^5 2e^{-st}dt + \int_5^T 2e^{-st}dt$$
$$=4[-\frac{1}{s}e^{-st}]_0^3 + 2[-\frac{1}{s}e^{-st}]_3^5 + 2[-\frac{1}{s}e^{-st}]_5^T$$
$$=\frac{4}{s}(1-e^{-3s})+\frac{2}{s}(e^{-3s}-e^{-5s})+\frac{2}{s}(e^{-5s}-e^{-sT})=\frac{2}{s}(2-e^{-3s}+e^{-5s})-\frac{2}{s}e^{-sT}$$
$$F(s)=\lim_{T\to\infty}\frac{2}{s}(2-e^{-3s}+e^{-5s})-\frac{2}{s}e^{-sT}=\frac{2}{s}(2-e^{-3s}+e^{-5s}) \quad , \quad \text{for } s>0$$

d)

$$F(s)=\int_0^3 e^{2t}\cdot e^{-st}dt+\int_3^\infty 0\,dt=\int_0^3 e^{(2-s)t}dt=[\frac{1}{2-s}e^{(2-s)t}]_0^3=\frac{1}{s-2}(1-e^{6-3s})$$

e)

$$\int_0^1 t\,e^{-st}dt+\int_1^T 2e^{-st}dt=[t\cdot(-\frac{1}{s}e^{-st})]_0^1-\int_0^1 -\frac{1}{s}\cdot e^{-st}dt+\int_1^T 2e^{-st}dt$$
$$=[t\cdot(-\frac{1}{s}e^{-st})]_0^1+[-\frac{1}{s^2}e^{-st}]_0^1+\int_1^T 2e^{-st}dt$$
$$=-\frac{1}{s}e^{-s}+[-\frac{1}{s^2}e^{-s}+\frac{1}{s^2}]+2[-\frac{1}{s}e^{-sT}]_0^T$$
$$F(s)=\frac{1}{s^2}-\frac{1}{s}e^{-s}-\frac{1}{s^2}e^{-s}+\lim_{T\to\infty}2[-\frac{1}{s}e^{-sT}]_0^T$$
$$=\frac{1}{s^2}-\frac{1}{s}e^{-s}-\frac{1}{s^2}e^{-s}+2\frac{1}{s}e^{-s}=\frac{1}{s^2}+\frac{1}{s}e^{-s}-\frac{1}{s^2}e^{-s}$$

f)

$$\int_0^2 0\cdot e^{-st}dt+\int_2^3 (t-2)e^{-st}dt+\int_3^T 1\,e^{-st}dt=\int_2^3 t\,e^{-st}dt+\int_2^3 (-2)e^{-st}dt+\int_3^T 1\,e^{-st}dt$$
$$=[t\cdot(-\frac{1}{s}e^{-st})]_2^3-\int_2^3 -\frac{1}{s}\cdot e^{-st}dt+\int_2^3 (-2)e^{-st}dt+\int_3^T 1\,e^{-st}dt$$
$$=[-\frac{t}{s}e^{-st}]_2^3-[\frac{1}{s^2}e^{-st}]_2^3+2[\frac{1}{s}e^{-st}]_2^3+[-\frac{1}{s}e^{-st}]_3^T dt$$
$$=-\frac{3}{s}e^{-3s}+\frac{2}{s}e^{-2s}-\frac{1}{s^2}e^{-3s}+\frac{1}{s^2}e^{-2s}+\frac{2}{s}e^{-3s}-\frac{2}{s}e^{-2s}+\frac{1}{s}e^{-3s}-\frac{1}{s}e^{-sT}$$
$$F(s)=\frac{1}{s^2}e^{-2s}-\frac{1}{s^2}e^{-3s}-\lim_{T\to\infty}\frac{1}{s}e^{-sT}$$
$$=\frac{1}{s^2}e^{-2s}-\frac{1}{s^2}e^{-3s}$$

14.3-2.bdf

b)

$$F(s)=3\frac{2!}{s^{2+1}}+5\frac{1}{s}=\frac{6}{s^3}+\frac{5}{s}$$

d)

$$F(s)=8\frac{3!}{s^{3+1}}+3\frac{1}{s--2}s=\frac{48}{s^4}+\frac{3}{s+2}$$

f)

$$F(s)=2\frac{s}{s^2+3^2}+4\frac{\sqrt{3}}{s^2+(\sqrt{3})^2}=\frac{2s}{s^2+9}+\frac{4\sqrt{3}}{s^2+3}$$

14.3-4.ab

a)

$$f'(t)=\begin{cases}1 & , \quad 0\le t<2\\ 0 & , \quad t\ge 2\end{cases}$$

$$L\{f'(t)\}=\int_0^2 1e^{-st}dt+\int_2^\infty 0\cdot e^{-st}dt=[-\frac{1}{s}e^{-st}]_0^2=\frac{1}{s}(1-e^{-2s})$$

b)

$$\begin{aligned} f(t)&=t\cdot[u(t-0)-u(t-2)]+2\cdot[u(t-2)-u(t-'\infty')]\\ &=t[\ 1 \qquad -u(t-2)]+2\cdot[u(t-2)- \qquad 0]\\ &=t-t\cdot u(t-2)+2\cdot u(t-2)\\ &=t-(t-2+2)\cdot u(t-2)+2\cdot u(t-2)\\ &=t-(t-2)\cdot u(t-2)-2\cdot u(t-2)+2\cdot u(t-2)\\ &=t-(t-2)\cdot u(t-2)\end{aligned}$$

$$F(s)=\frac{1}{s^2}-\frac{1}{s^2}e^{-2s}$$

$$L\{f'(t)\}=s\,F(s)-f(0)=s(\frac{1}{s^2}-\frac{1}{s^2}e^{-2s})-0=\frac{1}{s}-\frac{1}{s}e^{-2s}$$

14.3-5.abcd

a)

$$f(t)=t^3-2t^2+5 \quad , \quad f'(t)=3t^2-4t$$

$$\boldsymbol{L}(f')=s\,F(s)-f(0)=s(\frac{6}{s^4}-2\frac{2}{s^3}-\frac{5}{s})-0^3-2\cdot 0^2+5=\frac{6}{s^3}-\frac{4}{s^2}$$

$$\boldsymbol{L}(f'')=s^2F(s)-sf(0)-f'(0)=s^2(\frac{6}{s^4}-2\frac{2}{s^3}+\frac{5}{s})-s\cdot 5-0=\frac{6}{s^2}-\frac{4}{s}$$

b)

$$f(t)=\sin(2t) \quad , \quad f'(t)=2\cos(2t)$$

$$\boldsymbol{L}(f')=s\,F(s)-f(0)=s\frac{2}{s^2+2^2}-2\sin(2\cdot 0)=\frac{2s}{s^2+4}$$

$$\boldsymbol{L}(f'')=s^2F(s)-sf(0)-f'(0)=s^2\frac{2}{s^2+2^2}-s\cdot 0-2\cos(2\cdot 0)=\frac{2s^2}{s^2+4}-2\frac{s^2+4}{s^2+4}=\frac{-8}{s^2+4}$$

c)

$$f(t)=e^{3t} \quad , \quad f'(t)=3e^{3t}$$

$$\boldsymbol{L}(f')=sF(s)-f(0)=s\frac{1}{s-3}-e^0=\frac{s}{s-3}-\frac{s-3}{s-3}=\frac{3}{s-3}$$

$$\boldsymbol{L}(f'')=s^2F(s)-sf(0)-f'(0)=s^2\frac{1}{s-3}-se^0-3e^0=\frac{s^2}{s-3}-(s+3)\frac{s-3}{s-3}=\frac{9}{s-3}$$

d)

$$f(t)=4e^{-t}+5\cos(3t) \quad , \quad f'(t)=-4e^{-t}-15\sin(3t)$$

$$F(s)=\frac{4}{s+1}+\frac{5s}{s^2+3^2}$$

$$\boldsymbol{L}(f')=sF(s)-f(0)=s\frac{4}{s+1}+s\frac{5s}{s^2+9}-4-5$$

$$=s\frac{4}{s+1}-4\frac{s+1}{s+1}+s\frac{5s}{s^2+9}-5\frac{s^2+9}{s^2+9}=-\frac{4}{s+1}-\frac{45}{s^2+9}$$

$$\boldsymbol{L}(f'')=s^2F(s)-sf(0)-f'(0)=s^2\frac{4}{s+1}+s^2\frac{5s}{s^2+9}-9s-(-4)$$

$$=\frac{4s^2}{s+1}-(4s-4)\frac{s+1}{s+1}+\frac{5s^3}{s^2+9}-5s\frac{s^2+9}{s^2+9}$$

$$=\frac{4s^2-4s^2-44s+4s+4}{s+1}+\frac{5s^3-5s^3-45s}{s^2+9}=\frac{4}{s+1}-\frac{45s}{s^2+9}$$

14.3-6.ab

a)

$$f(t)=e^{-5t}$$

$$\boldsymbol{L}\left(\int_0^t f(\tau)d\tau\right)=\frac{1}{s}F(s)=\frac{1}{s}\frac{1}{s--5}=\frac{1}{s(s+5)}$$

b)

$$f(t)=\cos(5t)$$

$$\boldsymbol{L}\left(\int_0^t f(\tau)d\tau\right)=\frac{1}{s}F(s)=\frac{1}{s}\cdot\frac{s}{s^2+5^2}=\frac{1}{s^2+25}$$

14.3-7.abcde

a)

$$f(t)=t\cdot e^{3t}$$

$$F(s)=\frac{1!}{(s-3)^{1+1}}=\frac{1}{(s-3)^2}$$

b)

$$f(t)=t^2\cdot e^{4t}$$

$$F(s)=\frac{2!}{(s-4)^{2+1}}=\frac{2}{(s-4)^3}$$

c)

$$f(t)=t^4\cdot e^{-2t}$$

$$F(s)=\frac{4!}{(s--2)^{4+1}}=\frac{24}{(s+2)^5}$$

d)

$$f(t)=e^{2t}\sin(3t)$$
$$F(s)=\frac{3}{(s-2)^2+3^2}=\frac{3}{s^2-4s+13}$$

e)

$$f(t)=e^{-3t}\cos(5t)$$
$$F(s)=\frac{s--3}{(s--3)^2+5^2}=\frac{s+3}{s^2+6s+9+25}=\frac{s+3}{s^2+6s+34}$$

14.3-8.ab

a)

$$f(t)=t\cdot\sin(2t)=t\cdot g(t)$$
$$G(s)=\frac{2}{s^2+4}=2\cdot(s^2+4)^{-1}$$
$$F(s)=-\frac{dG(s)}{ds}=-(-2(s^2+4)^{-2}\cdot 2s)=\frac{4s}{(s^2+4)^{-2}}$$

b)

$$f(t)=t\cdot\cos(3t)=t\cdot g(t)$$
$$G(s)=\frac{s}{s^2+9}=s(s^2+9)^{-1}$$
$$F(s)=-\frac{dG(s)}{ds}=-(1(s^2+9)^{-1}+s\cdot(s^2+9)^{-2}\cdot 2s)=\frac{s^2-9}{(s^2+9)^2}$$

14.3-9.ab

a)

$$f(t)=\frac{e^{-t}-e^{-2t}}{t}$$
$$g(t)=e^{-t}-e^{-2t}$$
$$G(z)=\frac{1}{z+1}-\frac{1}{z+2}$$
$$F(s)=\int_s^\infty G(z)\,dz=\lim_{Z\to\infty}\int_s^Z \frac{1}{z+1}-\frac{1}{z+2}dz=\lim_{Z\to\infty}[\ln(z+1)-\ln(z+2)]_s^Z$$
$$=\lim_{Z\to\infty}(\ln\frac{Z+1}{Z+2}+\ln\frac{s+2}{s+1})=0+\ln\frac{s+2}{s+1}=\ln\frac{s+2}{s+1}$$

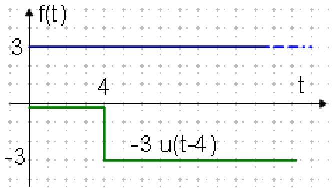
b)

$$f(t)=\frac{\sin t}{t}\quad,\quad g(t)=\sin t$$
$$G(z)=\frac{1}{z^2+1}$$
$$F(s)=\int_s^\infty G(z)\,dz=\lim_{Z\to\infty}\int_s^Z \frac{1}{z^2+1}dz=\lim_{Z\to\infty}[\tan^{-1}(z)]_s^Z$$
$$=\lim_{Z\to\infty}(\tan^{-1}(Z)-\tan^{-1}(s))=\frac{\pi}{2}-\tan^{-1}(s)$$

14.3-10.abcdefghi

a)

$$f(t)=3-3\cdot u(t-4)$$
$$F(s)=\frac{3}{s}-3\frac{1}{s}e^{-4t}=\frac{3}{s}(1-e^{-4t})$$

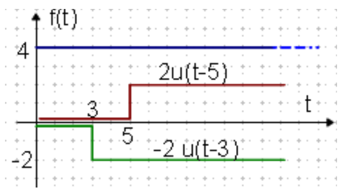


b)

$$f(t)=5\cdot u(t-2)-5\cdot u(t-4)$$
$$F(s)=5\frac{1}{s}e^{-2s}-5\frac{1}{s}e^{-4s}=\frac{5}{s}(e^{-2s}-e^{-4s})$$

c)

$$f(t)=4-2\cdot u(t-3)+2u(t-4)$$
$$F(s)=\frac{4}{s}-\frac{2}{s}e^{-3t}+\frac{2}{s}e^{-5t}=\frac{2}{s}(2-e^{-3s}+e^{-5s})$$

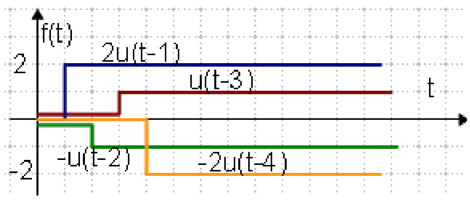


d)

$$f(t)=2\cdot u(t-1)-4\cdot u(t-2)+2\cdot u(t-3)$$
$$F(s)=2\frac{1}{s}e^{-s}-4\frac{1}{s}e^{-2s}+2\frac{1}{s}e^{-3s}=\frac{2}{s}(e^{-s}-2e^{-2s}+e^{-3s})$$

e)

$$f(t)=2u(t-1)-u(t-2)+u(t-3)-2u(t-4)$$
$$F(s)=\frac{2}{s}e^{-t}-\frac{1}{s}e^{-2t}+\frac{1}{s}e^{-3t}-\frac{2}{s}e^{-4t}=\frac{1}{s}(2e^{-t}-e^{-2t}+e^{-3t}-2)$$



f)

$$f(t)=2-3\cdot u(t-3)-u\cdot u(t-5)$$
$$F(s)=2\frac{1}{s}-3\frac{1}{s}e^{-3s}-\frac{1}{s}e^{-5s}=\frac{1}{s}(2-3e^{-3s}-e^{-5s})$$

g)

$$f(t)=\begin{cases} e^{2t} & \text{for } 0\le t<3 \\ 0 \;\text{ for} & t\ge 3 \end{cases}$$

Velger her å 'slå på' funksjonen e^{2t} ved å multiplisere med en '0-1-0-tidsluke' mellom *t*=0 og *t*=3.

$$f(t)=e^{2t}[u(t-0)-u(t-3)]=e^{2t}[1-u(t-3)]=e^{2t}-e^{2t}\cdot u(t-3)$$

Kombinasjonen av tidsfunksjon e^{2t} multiplisert med enhetssprang funksjon tilpasses samme tidsargument, *t*-3,

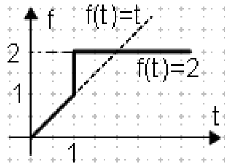
$$f(t) = e^{2t} - e^{2t-6+6}u(t-3) = e^{2t} - e^{2(t-3)+6}u(t-3) = e^{2t} + e^{6} \cdot e^{2(t-6)}u(t-3)$$

Det siste leddet i uttrykket har nå samme tidsskift argiment, *t*-3,

$$F(s) = \frac{1}{s-2} + e^{6}\frac{1}{s-2} \cdot e^{-3s} = \frac{1+e^{6-3s}}{s-2}$$

h)

$$f(t) = \begin{cases} t & \text{for } 0 \le 2 > 1 \\ 2 & \text{for } \quad t \ge 1 \end{cases}$$



Velger her å 'slå på' funksjonen $f(t) = t$ ved å multiplisere med en '0-1-0-tidsluke' mellom *t*=0 og *t*=1 og videre funksjonen $f(t) = 2$ med en tidsluke fra *t*=1 og utover.

$$\begin{aligned} f(t) &= t[u(t-0) - u(t-1)] + 2[u(t-1) - u(t - '\infty')] \\ &= t[1 - u(t-1)] + 2[u(t-1) - 0] \\ &= t - t \cdot u(t-1) + 2 \cdot u(t-1) \end{aligned}$$

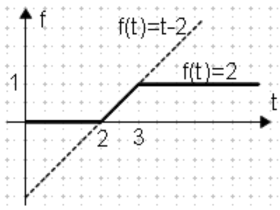
Kombinasjonen av tidsfunksjon t multiplisert med enhetssprang funksjon tilpasses samme tidsargument, *t*-1,

$$\begin{aligned} f(t) &= t + (-t + 1 - 1) \cdot u(t-1) + 2 \cdot u(t-1) \\ &= t - (t-1) \cdot u(t-1) - 1 \cdot u(t-1) + 2 \cdot u(t-1) \\ &= t - (t-1) \cdot u(t-1) + 1 \cdot u(t-1) \end{aligned}$$

$$F(s) = \frac{1}{s^2} - \frac{1}{s^2}e^{-s} + \frac{1}{s}e^{-s} = \frac{1 - e^{-s} + s e^{-s}}{s^2}$$

i)

$$f(t) = \begin{cases} 0 & \text{for} \quad 0 \le t < 2 \\ t-2 & \text{for} \quad 2 \le t < 3 \\ 1 & \text{for} \quad t \ge 3 \end{cases}$$



Velger her å 'slå på' funksjonen $f(t) = t-2$ ved å multiplisere med en '0-1-0-tidsluke' mellom *t*=2 og *t*=3 og videre funksjonen $f(t) = 1$ med en tidsluke fra *t*=3 og utover.

$$\begin{aligned} f(t) &= (t-2)[u(t-2) - u(t-3)] + 1[u(t-3) - u(t - '\infty')] \\ &= (t-2) \cdot u(t-2) - (t-2) \cdot u(t-3) + u(t-3) \\ &= (t-2) \cdot u(t-2) - (t-2 \mathbf{-1+1}) \cdot u(t-3) + u(t-3) \\ &= (t-2) \cdot u(t-2) - (t-3) \cdot u(t-3) - u(t-3) + u(t-3) \\ &= (t-2) \cdot u(t-2) - (t-3) \cdot u(t-3) \end{aligned}$$

$$F(s) = \frac{1}{s^2}e^{-2s} - \frac{1}{s^2}e^{-3s} = \frac{e^{-2s} - e^{-3s}}{s^2}$$

14.3-11.abcdefgh

a)

$$F(s) = \frac{6}{s^2+9} = 2\frac{3}{s^2+3^2} \qquad >> \qquad f(t) = 2\sin 3t$$

b)

$$F(s)=\frac{5}{s+3}-\frac{4}{s}=5\frac{1}{s-(-3)}-4\frac{1}{s} \qquad >> \qquad f(t)=5e^{-3t}-4$$

c)

$$F(s)=\frac{3s-2}{s^2+4}+\frac{9}{s^4}=3\frac{s}{s^2+2^2}-2\frac{1}{s^2+2^2}+\frac{9}{3!}\cdot\frac{3!}{s^4}$$

$$f(t)=3\cos 2t-2\sin 2t+\frac{3}{2}\cdot t^3$$

d)

$$F(s)=\frac{7}{(s-3)^5}=\frac{7}{4!}\cdot\frac{4!}{(s-3)^5} \qquad >> \qquad f(t)=\frac{7}{24}\cdot t^4 e^{3t}$$

e)

$$F(s)=\frac{s}{(s+1)^3}=\frac{s+1-1}{(s+1)^3}=\frac{1}{(s+1)^2}-\frac{1}{(s+1)^3}=\frac{1}{(s+1)^2}-\frac{1}{2}\frac{2}{(s+1)^3}$$

$$f(t)=t e^{-t}-\frac{1}{2}t^2 e^{-t}=(t-\frac{t^2}{2})e^{-t}$$

f)

$$F(s)=\frac{2s-6}{s^2+3}=2\frac{s}{s^2+(\sqrt{3})^2}-2\sqrt{3}\frac{\sqrt{3}}{s^2+(\sqrt{3})^2}$$

$$f(t)=2\cos(\sqrt{3}t)-2\sqrt{3}\sin(\sqrt{3}t)$$

g)

$$F(s)=\frac{7}{s^2+2s+2}=\frac{7}{s^2+2s+1+1}=7\frac{1}{(s+1)^2+1}$$

$$f(t)=7e^{-t}\sin t$$

h)

$$F(s)=\frac{2s-1}{s^2-4s+13}=\frac{2s-1}{s^2-4s+4+9}=\frac{2s-1}{(s-2)^2+3^2}$$
$$=\frac{2s-1+3-3}{(s-2)^2+3^2}=\frac{2(s-2)+3}{(s-2)^2+3^2}=2\frac{s-2}{(s-2)^2+3^2}+\frac{3}{(s-2)^2+3^2}$$

$$f(t)=2e^{2t}\cos(3t)+e^{2t}\sin(3t)$$

14.3-12.abcdefgh

a)

$$F(s)=\frac{e^{-s}+e^{-3s}}{s}=e^{-s}\frac{1}{s}+e^{-3s}\frac{1}{s} \qquad \left(\boldsymbol{L}^{-1}\{\frac{1}{s}\}=1 \right)$$

$$f(t)=u(t-1)+u(t-3)=\begin{cases}0 & \text{for} \quad 0\le t<1\\ 1 & \text{for} \quad 1\le t<3\\ 2 & \text{for} \quad t\ge 3\end{cases}$$

b)

$$F(s)=\frac{e^{-5s}}{s^2}=e^{-5s}\frac{1}{s^2} \qquad \left(\boldsymbol{L}^{-1}\{\frac{1}{s^2}\}=t \right)$$

$$f(t)=(t-5)\cdot u(t-5)=\begin{cases} 0 & \text{for} \quad 0\le t<5 \\ t-5 & \text{for} \quad t\ge 5 \end{cases}$$

c)

$$F(s)=\frac{e^{-3s}}{s-1}=e^{-3s}\frac{1}{s-1} \qquad \left(\boldsymbol{L}^{-1}\{\frac{1}{s-1}\}=e^{t} \right)$$

$$f(t)=e^{t-3}\cdot u(t-3)=\begin{cases} 0 & \text{for} \quad 0\le t<3 \\ e^{t-3} & \text{for} \quad t\ge 3 \end{cases}$$

d)

$$F(s)=\frac{e^{-4s}}{(s+3)^2}=e^{-4s}\frac{1}{(s+3)^2} \qquad \left(\boldsymbol{L}^{-1}\{\frac{1}{(s+3)^2}\}=t\cdot e^{-3t} \right)$$

$$f(t)=(t-4)\cdot e^{-3(t-4)}\cdot u(t-4)=\begin{cases} 0 & \text{for} \quad 0\le t<4 \\ (t-4)e^{-3(t-4)} & \text{for} \quad t\ge 3 \end{cases}$$

e)

$$F(s)=\frac{s}{s^2+1}\cdot e^{-\pi s} \qquad \left(\boldsymbol{L}^{-1}\{\frac{s}{s^2+1}\}=\cos t \right)$$

$$f(t)=\cos(t-\pi)\cdot u(t-\pi)=\begin{cases} 0 & \text{for} \quad 0\le t<\pi \\ \cos(t-\pi) & \text{for} \quad t\ge \pi \end{cases}$$

Funksjonen kan forenkles videre fordi $\cos(\alpha-\pi)=-\cos\alpha$,

$$f(t)=\begin{cases} 0 & \text{for} \quad 0\le t<\pi \\ -\cos t & \text{for} \quad t\ge \pi \end{cases}$$

f)

$$F(s)=\frac{e^{-2\pi s}}{s^2+4}=\frac{1}{2}\frac{2}{s^2+2^2}e^{-2\pi s} \qquad \left(\boldsymbol{L}^{-1}\{\frac{2}{s^2+2^2}\}=\sin(2t) \right)$$

$$f(t)=\frac{1}{2}\sin(2(t-2\pi))\cdot u(t-2\pi)=\begin{cases} 0 & \text{for} \quad 0\le t<2\pi \\ \frac{1}{2}\sin(2t-4\pi) & \text{for} \quad t\ge 2\pi \end{cases}$$

Funksjonen kan forenkles videre fordi $\sin(\alpha-4\pi)=\sin\alpha$,

$$f(t)=\begin{cases} 0 & \text{for} \quad 0\le t<2\pi \\ \frac{1}{2}\sin(2t) & \text{for} \quad t\ge 2\pi \end{cases}$$

g)

$$F(s)=\frac{e^{-3s}}{s^2+4s+5}=\frac{1}{s^2+4s+4+1}\cdot e^{-3s}=\frac{1}{(s+2)^2+1^2}\cdot e^{-3s} \qquad \left(\boldsymbol{L}^{-1}\{\frac{1}{(s+2)^2+1^2}\}=e^{-2t}\sin t \right.$$

$$f(t)=[e^{-2(t-3)}\sin(t-3)]\cdot u(t-3)=\begin{cases} 0 & \text{for} \quad 0\le t<3 \\ e^{6-2t}\sin(t-3) & \text{for} \quad t\ge 3 \end{cases}$$

h)

$$F(s)=\frac{e^{-(s-3)}}{s-3}=e^3\frac{1}{s-3}e^{-s} \qquad \left(\ \boldsymbol{L}^{-1}\{\frac{1}{s-3}\}=e^{3t}\ \right)$$

$$f(t)=[e^3\cdot e^{3(t-1)}]\cdot u(t-1)=e^{3t}\cdot u(t-1)=\begin{cases}0 & \text{for} \quad 0\le t<1\\ e^{3t} & \text{for} \quad t\ge 1\end{cases}$$

14.3-13.abcd

a)

$$F(s)=\frac{s-13}{s^2-5x+4}=\frac{s-13}{(s-1)(s-4)}=\frac{A}{s-1}+\frac{B}{s-4}=\frac{(A+B)s-4A-B}{s^2-5x+4}$$

$$\begin{matrix}A+B=1\\ -4A-B=-13\end{matrix} \quad \Rightarrow \quad \begin{matrix}A=4\\ B=-3\end{matrix}$$

$$F(s)=\frac{s-13}{s^2-5x+4}=\frac{4}{s-1}-\frac{3}{s-4} \quad \Rightarrow \quad f(t)=4e^t-3e^{4t} \quad , \quad t\ge 0$$

b)

$$F(s)=\frac{5s+2}{s^3+2s^2}=\frac{A}{s^2}+\frac{B}{s}+\frac{C}{s+2}=\frac{(B+C)s^2+(A+2B)s+2A}{s^3+2s^2}$$

$$\begin{matrix}B+C=0\\ A+2B=5\\ 2A=2\end{matrix} \quad \Rightarrow \quad \begin{matrix}A=1\\ B=2\\ C=-2\end{matrix}$$

$$F(s)=\frac{5s+2}{s^3+2s^2}=\frac{1}{s^2}+\frac{2}{s}-\frac{2}{s+2} \quad \Rightarrow \quad f(t)=t+2-2e^{-2t} \quad , \quad t\ge 0$$

c)

```
MATLAB/Octave numerisk
>> % Legger polynomfaktorene inn i radvektorer
>> teller = [1 2 23 19];
>> nevnerS = [1 1 -2];        % Denne skal opphøyes i 2.
>> nevner = conv(nevnerS, nevnerS);
>> % Utfører delbrøkspalting
>> [rot, pol] = residue(teller, nevner)
>> rot =    1  -3   0   5
>> pol =   -2  -2   1   1
```

$$F(s)=\frac{s^3+2s^2+23s+19}{(s^2+s-2)^2}=\frac{1}{s+2}-\frac{3}{(s+2)^2}+\frac{5}{(s-1)^2} \quad \Rightarrow \quad f(t)=e^{-2t}-3te^{-2t}+5te^t$$

14.3-14.abc

a)

Differensiallikning $\quad y'-5y=0 \quad , \quad y(0)=4$

Vi har generelt $\quad L\{y\}=Y \quad , \quad L\{y'\}=sY-y(0)$

Laplacetransform av likning: $\quad sY-y(0)-5Y=0$

$$sY-4-5Y=0$$

$$Y=\frac{4}{s-5}$$

Invers laplacetransform, $\quad y(t)=4e^{5t}$

b)

Differensiallikning $y'+2y=8 \quad , \quad y(0)=3$

Vi har generelt $L\{y\}=Y \quad , \quad L\{y'\}=sY-y(0)$

Laplacetransform av likning: $sY-y(0)+2Y=8\frac{1}{s}$

$$sY-3+2Y=\frac{8}{s}$$

$$Y=\frac{\frac{8}{s}+3}{s+2}=\frac{8}{s(s+2)}+\frac{3}{s+2}=\frac{4}{s}-\frac{4}{s+2}+\frac{3}{s+2}$$

Invers laplacetransform, $y(t)=4-e^{-2t}$

c)

Differensiallikning $y'-3y=12t^2-8t-3 \quad , \quad y(0)=2$

Vi har generelt $L\{y\}=Y \quad , \quad L\{y'\}=sY-y(0)$

Laplacetransform av likning: $sY-2-3Y=12\frac{2}{s^3}-8\frac{1}{s^2}-3\frac{1}{s}$

$$Y(s-3)=\frac{24}{s^3}-\frac{8}{s^2}-\frac{3}{s}+2$$

$$Y=\frac{24}{s^3(s-3)}-\frac{8}{s^2(s-3)}-\frac{3}{s(s-3)}+\frac{2}{s-3}$$
$$=\frac{A}{s^3}+\frac{B}{s^2}+\frac{C}{s}+\frac{D}{s-3}=\frac{-8}{s^3}+\frac{1}{s}+\frac{1}{s-3}$$

Invers laplacetransform, $y(t)=-4t^2+e^{3t}+1$

14.3-15.ab

a)

Differensiallikning $y''-6y'+5y=0 \quad , \quad y(0)=0 \quad , \quad y'(0)=4$

Vi har generelt $L\{y\}=Y \quad , \quad L\{y'\}=sY-y(0)$
$L\{y''\}=s^2Y-s\,y(0)-y'(0)$

Laplacetransform av likning: $s^2Y-s\cdot 0-4-6(sY-0)+5Y=0$

$$s^2Y-6sY+5Y=4$$

$$Y=\frac{4}{s^2-6s+5}=\frac{4}{(s-1)(s-5)}=\frac{1}{s-5}-\frac{1}{s-1}$$

Invers laplacetransform, $y(t)=e^{5t}-e^{t}$

b)

Differensiallikning $y''+6y'+8y=0 \quad , \quad y(0)=2 \quad , \quad y'(0)=-8$

Vi har generelt $L\{y\}=Y \quad , \quad L\{y'\}=sY-y(0)$
$L\{y''\}=s^2Y-s\,y(0)-y'(0)$

Laplacetransform av likning: $s^2Y-s\cdot 2+8+6(sY-2)+8Y=0$

$$s^2Y+6sY+8Y=2s+4$$

$$Y=\frac{2s+4}{s^2+6s+8}=\frac{2s+4}{(s+2)(s+4)}=\frac{2}{s+4}$$

Invers laplacetransform, $y(t)=2e^{-4t}$

14.3-16.ab

a)

$$f(t)=\begin{cases} 5 & \text{for} \quad 0\le t<3 \\ 0 & \text{for} \quad t\ge 3 \end{cases} \qquad f(t)=5-5\cdot u(t-3)$$

$$F(s)=\frac{5}{s}-\frac{5}{s}e^{-3s}$$

b)

Differensiallikning $\quad y'-5y=5-5\cdot u(t-3)\ ,\quad y(0)=1$

Vi har generelt $\quad L\{y\}=Y\ ,\quad L\{y'\}=sY-y(0)$

Laplacetransform av likning: $\quad sY-1-5Y=\frac{5}{s}-\frac{5}{s}e^{-3s}$

$$Y(s-5)=1+\frac{5}{s}-\frac{5}{s}e^{-3s}$$

$$\begin{aligned} Y&=\frac{1}{s-5}+\frac{5}{s(s-5)}-\frac{5}{s(s-5)}e^{-3s} \\ &=\frac{1}{s-5}+\frac{A}{s}+\frac{B}{s-5}-\left(\frac{A}{s}+\frac{B}{s-5}\right)e^{-3s} \\ &=\frac{1}{s-5}-\frac{1}{s}+\frac{1}{s-5}-\left(-\frac{1}{s}+\frac{1}{s-5}\right)e^{-3s} \\ &=-\frac{1}{s}+2\frac{1}{s-5}-\left(\frac{1}{s}-\frac{1}{s-5}\right)e^{-3s} \end{aligned}$$

Invers laplacetransform,

$$\begin{aligned} y(t)&=-1+2e^{5t}+1\cdot u(t-3)-e^{5(t-3)}\cdot u(t-3) \\ &=-1+1\cdot u(t-3) \quad + \quad 2e^{5t}-e^{5(t-3)}\cdot u(t-3) \end{aligned}$$

Kan settes som stykkevis definert funksjon,

$$y_a(t)=-1+1\cdot u(t-3)=\begin{cases} -1 & \text{for} \quad 0\le t<3 \\ 0 & \text{for} \quad t\ge 3 \end{cases}$$

$$y_b(t)=2e^{5t}-e^{5(t-3)}\cdot u(t-3)=\begin{cases} 2e^{5t} & \text{for} \quad 0\le t<3 \\ 2e^{5t}-e^{5t-15} & \text{for} \quad t\ge 3 \end{cases}$$

$$y(t)=\begin{cases} 2e^{5t}-1 & \text{for} \quad 0\le t<3 \\ 2e^{5t}-e^{5t-15} & \text{for} \quad t\ge 3 \end{cases}$$

14.3-18.b

Differensiallikninger

$$\begin{aligned} y_1'&=2y_1-y_2\ ,\quad y_1(0)=2 \\ y_2'&=5y_1-2y_2\ ,\quad y_2(0)=3 \end{aligned}$$

Vi har generelt

$L\{y\}=Y\ ,\quad L\{y'\}=sY-y(0)$

Laplacetransform av likninge
$$sY_1-2=2Y_1-Y_2$$
$$sY_2-3=5Y_1-2Y_2$$

Ordner
$$(s-2)Y_1+Y_2=2$$
$$-5Y_1+(s+2)Y_2=3$$

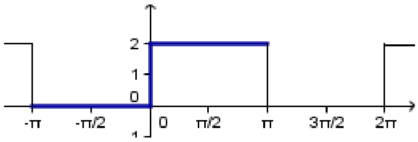
Velger å finne Y_2 av øverste likning og sette inn i nederste,
$$Y_2=2-(s-2)Y_1$$
$$-5Y_1+(s+2)(2-(s-2)Y_1)=3$$
$$Y_1=\frac{2s+1}{s^2+1}$$

- og løser for Y_2
$$Y_2=2-\frac{(s-2)(2s+1)}{s^2+1}=\frac{2s^2+2}{s^2+1}-\frac{(s-2)(2s+1)}{s^2+1}$$
$$=\frac{3s+4}{s^2+1}$$

Invers laplacetransform, $Y_1=2\cos t+\sin t$ $\quad Y_2=3\cos t+4\sin t$

14.4-6.abc

a)



Periode $T=2\pi$

Vi ser på 'øyemål' at middelverdien er 1 og $a_0=\text{'Dobbel middelverdi'}=2$,

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx=\frac{1}{\pi}\left(\int_{-\pi}^{0} 0\,dx+\int_{0}^{\pi} 2\,dx\right)=\frac{2}{\pi}[x]_0^{\pi}=\frac{2}{\pi}(\pi-0)=2$$

Funksjonen har ikke direkte symmetriegenskaper som like eller odde funksjon, men om vi 'tar bort' middelverdien får vi en origosymmetrisk graf som veksler mellom verdiene -1 og +1. Etter regelen om fourierkoeffisienter for sum av to funksjoner har vi da at fourierrekka består av konstantledd a_0 og sinuskoeffisienter b_n .

Koeffisientene b_n beregnes med *n* som parameter,

$$b_n=\frac{2}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\sin(nx)\,dx$$
$$=\frac{2}{\pi}\left(\int_{-\pi}^{0} 0\cdot\sin(nx)\,dx+\int_{0}^{\pi} 1\cdot\sin(nx)\,dx\right)$$
$$=\frac{2}{\pi}\left[-\frac{1}{n}\cos(nx)\right]_0^{\pi}=\frac{2}{n\pi}(1-\cos(n\pi))$$

Setter vi så *n*=1, 2, 3, 4, ... vil vi finne at verdiene for b_n blir

$$b_n=\frac{4}{\pi},0,\frac{4}{3\pi},0,\frac{4}{5\pi},0,\frac{4}{7\pi},...=\frac{4}{(2n-1)\pi} \quad , \quad n=1,2,3,...$$

Fourierrekka alt i alt:

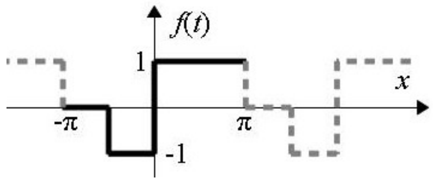
$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$$
$$=1+\sum_{m=1}^{\infty} \frac{4}{(2m-1)\pi}\sin((2m-1)x)=1+\frac{4}{\pi}\sum_{m=1}^{\infty}\frac{1}{2m-1}\sin((2m-1)x)$$

b)

Denne funksjonen veksler mellom +1 og +3 og har en middelverdi på +2, og fourierrekka blir den samme som i 14.4-6.b – bortsett fra konstantleddet,

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n \cos(nx)+b_n \sin(nx)$$
$$=2+\sum_{m=1}^{\infty} \frac{4}{(2m-1)\pi}\sin((2m-1)x)=1+\frac{4}{\pi}\sum_{m=1}^{\infty}\frac{1}{2m-1}\sin((2m-1)x)$$

c)



Får ikke hjelp av symmetriegenskaper her.

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx=\frac{1}{\pi}\left(\int_{-\pi}^{-\pi/2} 0\,dx+\int_{-\pi/2}^{0} -1\,dx+\int_{0}^{\pi} 1\,dx\right)$$
$$=\frac{1}{\pi}\left([-x]_{-\pi/2}^{0}+[x]_0^{\pi}\right)=\frac{1}{\pi}(-\pi/2+\pi)=\frac{1}{2}$$

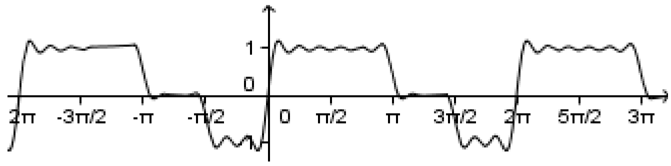
$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\cos nx dx=\frac{1}{\pi}\left(\int_{-\pi}^{-\pi/2} 0\cdot\cos nx\,dx+\int_{-\pi/2}^{0} -1\cdot\cos nx\,dx+\int_{0}^{\pi} 1\cdot\cos nx\,dx\right)$$
$$=\frac{1}{\pi}\left(\frac{1}{n}[-\sin nx]_{-\pi/2}^{0}+\frac{1}{n}[\sin nx]_0^{\pi}\right)$$
$$=\frac{1}{n\pi}\left(0+\sin\frac{-n\pi}{2}+\sin n\pi-0\right)=-\frac{1}{n\pi}\left(\sin\frac{n\pi}{2}\right) \qquad =\frac{1}{n\pi}(-1,0,1,0,-1,0,1,0,\ldots)$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cdot\sin nx dx=\frac{1}{\pi}\left(\int_{-\pi}^{-\pi/2} 0\cdot\sin nx\,dx+\int_{-\pi/2}^{0} -1\cdot\sin nx\,dx+\int_{0}^{\pi} 1\cdot\sin nx\,dx\right)$$
$$=\frac{1}{\pi}\left(\frac{1}{n}[\cos nx]_{-\pi/2}^{0}+\frac{1}{n}[-\cos nx]_0^{\pi}\right)$$
$$=\frac{1}{n\pi}\left(1-\cos\frac{-n\pi}{2}-\cos n\pi+1\right)$$
$$=\frac{1}{n\pi}\left(2-\cos\frac{n\pi}{2}-\cos n\pi\right) \qquad =\frac{1}{n\pi}(3,2,3,0,3,2,3,0,\ldots)$$

Fourierrekka blir

$$\frac{1}{2}\boldsymbol{a_0}+\boldsymbol{a_1}\cos x+\boldsymbol{b_1}\sin x+\boldsymbol{a_2}\cos 2x+\boldsymbol{b_2}\sin 2x+\boldsymbol{a_3}\cos 3x+\boldsymbol{b_3}\sin 3x+\ldots$$
$$\frac{1}{4}+\frac{1}{\pi}\left(-\cos x+3\sin x+0\cdot\cos 2x+\frac{2}{2}\sin 2x+\frac{1}{3}\cos 3x+\frac{3}{3}\sin 3x+0\cdot\cos 4x+0\sin 4x+\ldots\right)$$

Tester svaret med en graf for noen ledd,



Det generelle uttrykket for fourierrekka er litt innfløkt på grunn av den noe uregelmessige vekslingen i teller for brøkene som gir koeffisientene.

Tallfølgen $-1, 0, 1, 0, -1, 0, 1, 0, \ldots$ kan dannes av $(-1)^k$ for $k=1, 2, 3, \ldots$ og samtidig bruke $m=2k-1$ som nevner og frekvensmultiplikator.

Tallfølgen $3, 2, 3, 0, 3, 2, 3, 0, \ldots$ er ikke helt grei, men den kan dannes som summen av leddene i tallfølgene $3, 1, 3, 1, 3, 1, 3, 1, \ldots$ og $0, 1, 0, -1, 0, 1, 0, -1, \ldots$ på følgende måte, $c=2-(-1)^k-\cos(k\frac{\pi}{2})$ for $k=1, 2, 3, \ldots$.

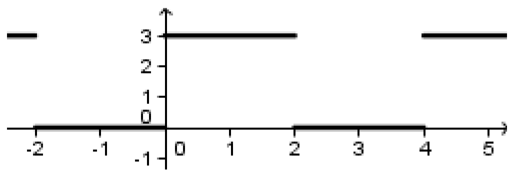
Fourierrekka på generell form,

$$\frac{1}{2}\boldsymbol{a_0}+\sum_{n=1}^{\infty}\boldsymbol{a_n}\cos(nx)+\boldsymbol{b_n}\sin(nx)$$
$$=\frac{1}{4}+\frac{1}{\pi}\sum_{k=1}^{\infty}(-1)^k\frac{1}{2k-1}\cos((2k-1)x) \;+\; \frac{1}{\pi}\sum_{m=1}^{\infty}\frac{2-(-1)^m-\cos(m\pi/2)}{m}\cdot\sin(mx)$$

Her må det brukes ulike indeksvariabler, *k* og *m*. Som alternativ kan uttrykkene settes etter samme summetegn med sinus- og cosinusleddene som lager vekslingen i tallfølgene,

$$\frac{1}{2}\boldsymbol{a_0}+\sum_{n=1}^{\infty}\boldsymbol{a_n}\cos(nx)+\boldsymbol{b_n}\sin(nx)$$
$$=\frac{1}{4}+\frac{1}{\pi}\sum_{k=1}^{\infty}\frac{1}{n}\left[\left(-\sin\frac{n\pi}{2}\right)\cos nx \;+\; \left(2-\cos\frac{n\pi}{2}-\cos n\pi\right)\sin nx\right]$$

14.4-9.a



Middelverdi: $\frac{3}{2}$

Periode: $T=2\cdot L=2\cdot 2$

Fourierrekka vil inneholde bare a_0 og b_n ledd fordi funksjonen er origosymmetrisk hvis middelverdien trekkes fra,

$$a_0=\frac{2}{L}\int_0^L f(x)dx=\frac{2}{2}\int_0^2 3\,dx=[3x]_0^2=3$$

$$b_n=\frac{2}{L}\int_0^L (f(x)-\frac{a_0}{2})\sin(\frac{n\pi}{L}x)dx=\frac{2}{2}\int_0^2\frac{3}{2}\sin(\frac{n\pi}{2}x)dx$$
$$=\frac{3}{2}\cdot\frac{2}{n\pi}[-\cos(\frac{n\pi}{2}x)]_0^2=\frac{3}{n\pi}(\cos(n\pi)+1) \qquad =\frac{3}{n\pi}(0,2,0,2,0,\ldots)=\frac{6}{n\pi}(0,1,0,1,0,\ldots)$$

Fourierrekka blir

$$\frac{1}{2}a_0+\sum_{n=1}^{\infty}b_n\sin(\frac{n\pi}{L}x)$$
$$=\frac{3}{2}+\frac{6}{\pi}\sum_{k=1}^{\infty}\frac{1}{2k-1}\sin(\frac{(2k-1)\pi}{2}x)$$

14.4-12.abc

a) $f(-x) \neq f(x)$ og $-f(-x) \neq f(x)$, ingen symmetri.

b) $f(-x) = f(x)$ og $-f(-x) \neq f(x)$, *y*-akse symmetri, like funksjon.

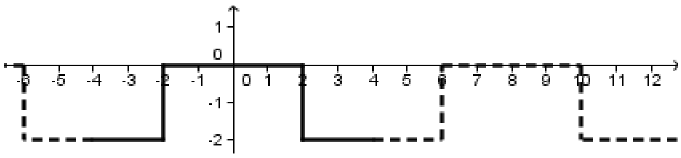
c) $f(-x) \neq f(x)$ og $-f(-x) = f(x)$, origo symmetri, odde funksjon.

14.4-13.ab

a)

Fourierrekka vil bare bestå av middelverdileddet, $\frac{a_0}{2} = 5$

b)



Middelverdi: -1
Periode: $T = 2 \cdot L = 2 \cdot 4$
y-akse symmetri, like funksjon

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx = \frac{2}{4}\left[\int_0^2 0\,dx + \int_2^4 -2\,dx\right] = [-x]_2^4 = -2$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi}{L}x\right)dx = \frac{2}{4}\left[\int_0^2 0\cos\left(\frac{n\pi}{4}x\right)dx + \int_2^4 -2\cos\left(\frac{n\pi}{4}x\right)dx\right]$$

$$= \frac{4}{4}\cdot\frac{4}{n\pi}\left[-\sin\left(\frac{n\pi}{4}x\right)\right]_2^4 = \frac{4}{n\pi}\left(-\sin(n\pi) + \sin\left(\frac{n\pi}{2}\right)\right) \qquad = \frac{4}{n\pi}(1, 0, -1, 0, 1, \ldots)$$

Fourierrekka blir

$$\frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{L}x\right)$$

$$= -1 + \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{2k-1}\cos\left(\frac{(2k-1)\pi}{4}x\right)$$

14.4-14.ab

a)

```
                ___                         ___
      _____|    |_________________|    |________  ->t
 -4    -3    -2    -1    0    1    2    3    4    5
```

Like utvidelse (til y-akse symmetri)
Fourierrekka vil bestå av middelverdi ($a_0/2$) og cosinus-ledd med amplitudefaktorer a_n

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx = \frac{2}{3}\left(\int_0^2 0\,dx + \int_2^3 2\,dx\right) = \frac{2}{3}[2x]_2^3 = \frac{4}{3}$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi}{L}x\right)dx = \frac{2}{3}\int_2^3 2\cos\left(\frac{n\pi}{3}x\right)dx$$

$$= \frac{2}{3}\cdot\frac{3}{n\pi}\left[\sin\left(\frac{n\pi}{3}x\right)\right]_2^3 = \frac{2}{n\pi}\sin(n\pi) - \frac{2}{n\pi}\sin\left(n\cdot\frac{2\pi}{3}\right) = 0 - \frac{2}{n\pi}\sin\left(n\cdot\frac{2\pi}{3}\right)$$

Setter vi inn *n*=1, 2, 3, 4, ... får vi

$$a_n = \{-\frac{\sqrt{3}}{\pi}, \frac{\sqrt{3}}{\pi}, 0, -\frac{\sqrt{3}}{\pi}, \frac{\sqrt{3}}{\pi}, 0, \ldots\}$$

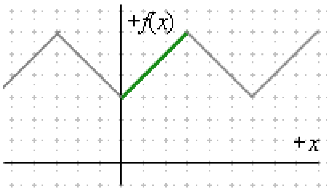
- som ikke er helt enkelt å generalisere. Derfor beholdes det generelle uttrykket

$$a_n = -\frac{2}{n\pi}\sin(n\cdot\frac{2\pi}{3})$$

Fourierrekka blir

$$\frac{2}{3} - \frac{2}{n\pi}\sin(n\cdot\frac{2\pi}{3})\cdot\cos(\frac{n\pi}{3}x)$$

b)



Like utvidelse (til y-akse symmetri)
Fourierrekka vil bestå av middelverdi ($a_0/2$)
og cosinus-ledd med amplitudefaktorer a_n

$$a_0 = \frac{2}{L}\int_0^L f(x)\,dx = \frac{2}{3}\int_0^3 x+3\,dx = \frac{2}{3}[\frac{1}{2}x^2+3x]_0^3 = 9$$

(Med en så enkel kurveform som dette 'ser vi lett at' middelverdien er 9/2.)

$$a_n = \frac{2}{L}\int_0^L f(x)\cos(\frac{n\pi}{L}x)\,dx = \frac{2}{3}\int_0^3 (x+3)\cos(\frac{n\pi}{3}x)\,dx$$
$$= [\frac{2}{n\pi}\cdot x\cdot\sin(\frac{n\pi}{3}x) + \frac{6}{n\pi}\sin(\frac{n\pi}{3}x) + \frac{6}{n^2\pi^2}\cos(\frac{n\pi}{3}x)]_0^3$$

Setter vi inn *n*=1, 2, 3, 4, ... får vi

$$a_n = \{-\frac{12}{\pi^2}, 0, -\frac{12}{3^2\pi^2}, 0, -\frac{12}{5^2\pi^2}, 0, -\frac{12}{7^2\pi^2}, 0, \ldots\}$$

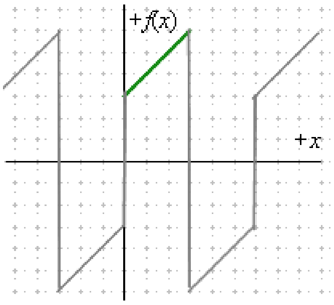
som på generell form blir

$$a_n = -\frac{12}{(2m-1)^2\pi^2} \qquad m = 1, 2, 3, \ldots$$

Fourierrekka blir

$$\frac{9}{2} - \frac{12}{\pi^2}\sum_{m=1}^{\infty}\frac{1}{(2m-1)^2}\cos(\frac{(2m-1)\pi}{3}x) \qquad m = 1, 2, 3, \ldots$$

14.4-15.b



Odde utvidelse (til origo symmetri)
Fourierrekka vil bestå av bare
sinusledd med amplitudefaktorer b_n

$$b_n=\frac{2}{L}\int_0^L f(x)\sin(\frac{n\pi}{L}x)dx=\frac{2}{3}\int_0^3 (x+3)\sin(\frac{n\pi}{3}x)dx$$

$$=[-\frac{2}{n\pi}\cdot x\cdot\cos(\frac{n\pi}{3}x)-\frac{6}{n\pi}\cos(\frac{n\pi}{3}x)+\frac{6}{n^2\pi^2}\sin(\frac{n\pi}{3}x)]_0^3$$

Setter vi inn *n*=1, 2, 3, 4, ... får vi

$$b_n=\{\frac{18}{\pi},-\frac{6}{2\pi},\frac{18}{3\pi}-\frac{6}{4\pi},\frac{18}{5\pi},...\}$$

som på generell form blir

$$b_n=\frac{6}{\pi}\frac{1-2\cdot(-1)^n}{n} \qquad n=1,2,3,...$$

Fourierrekka blir

$$\frac{6}{\pi}\sum_{n=1}^{\infty}\frac{1-2\cdot(-1)^n}{n}\sin(\frac{n\pi}{3}x) \qquad n=1,2,3,...$$