Combining the partial copula with quantile regression to test conditional independence

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Conditional independence testing lies at the heart of causal graphical structure learning due to its use in constraint-based structure learning algorithms such as the PC and FCI algorithms. One of the most common ways of testing conditional independence of X and Y given Z is by testing for vanishing partial correlation, where the partial correlation can be computed as the correlation between residuals after performing conditional mean regression.

In this talk we consider testing conditional independence by using a different residualization approach. More specifically, we residualize by transforming the variables by their conditional distribution functions, $U_1 = F(X \mid Z)$ and $U_2 = F(Y \mid Z)$, and then test for independence between U_1 and U_2 using a generalized correlation measure. Carrying out the test in practice requires estimation of the conditional distribution functions, and we present an estimator of F based on conditional quantile regression for which we can quantify the consistency rate. Furthermore, we show how the consistency rate of the conditional distribution function estimator can be transferred to level and power properties of the conditional independence test.

Lastly, we discuss the benefits of using this residualization approach over conventional residuals, and we present simulations which demonstrates that our test has superior power in cases with conditional variance heterogeneity.

References

[1] Petersen, L., & Hansen, N. R. (2021). Testing Conditional Independence via Quantile Regression Based Partial Copulas. *Journal of Machine Learning Research*, 22(70), 1-47.