Numerical cubature with symmetry and applications to polynomial optimisation

This PhD is funded by the Marie Curie program of European Union through the innovative training network (ITN) POEMA on polynomial optimization.

More info and positions at https://easychair.org/cfp/POEMA-19-22.

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Scientific context. Cubature rules are used in numerical analysis to compute integrals with respect to a given measure by approximating the definite integral of a function by a positively weighted sum of evaluations at specified points, or nodes, within the domain of integration. For a given measure and a fixed strength d one aims to specify the nodes and weights in such a way that the resulting quadrature rule is exact for all polynomials of degree at most d. For the case of measures on the real line it is classically known that d nodes can be always chosen in a way to exactly approximate a definite integral up to degree $2d-1$. However, in the general case of cubature in n dimensions the minimal number of nodes is not known and there exist only upper bounds (see [2]). The main objective of this project will be to construct cubature rules with a number of nodes near to the known bound and study to what extend this number is optimal. Following the classical results it seems reasonable to expect that such examples will be found amongst symmetric cubature rules, i.e., cubature rules for measures that are invariant by the action of a group G. This situation has in particular been studied recently by Collowald and Hubert [1], who provided a systematic way to construct symmetric cubature rules using moment-matrices combined with block-diagonalization techniques. The doctoral student will combine these techniques and theoretical approaches to construct symmetric cubature rules with a low number of nodes. More generally, it will be discussed how the results and methods then can be used in the more general framework of tensor decomposition. Furthermore, development of experimentation for industrial applications will be explored.

Working Context. The PhD candidate will be hosted by the Algebra research group and the SymRAG (Symmetry in Quantitative and Algorithmic Real Algebraic Geometry) research project at the Institute of Mathematics and Statistics at UIT. The team, led by Cordian Riener, has a strong expertise in effective real algebraic geometry, polynomial optimization, and coding theory.

Planned secondments. The PhD candidate will have research stays (secondments) at INRIA Sophia Antipolis (Evelyne Hubert), University of Konstanz (Markus Schweighofer), and ARTELYS (Arnaud Renaud).

Required Skills. Motivated candidate should hold — at the date of recruitment — a Master's degree in Computer Science, Mathematics or Engineering (or any equivalent

diploma). The applicant should have a solid background in either optimization, real algebraic geometry or computer algebra. Good programming skills are also a plus. Knowledge of Norwegian does not constitute a pre-requisite.

The candidates are kindly asked to send an e-mail with "POEMA candidate" in the title, a CV and motivation letter to cordian.riener@uit.no and to submit their documents at https://easychair.org/cfp/POEMA-19-22.

References

[1] M. Collowald, E. Hubert. Algorithms for computing cubatures based on moment theory. Studies in Applied Mathematics (141), pp. 501–546, 2018.

[2] C. Riener, M. Schweighofer. Optimization approaches to quadrature: New characterizations of Gaussian quadrature on the line and quadrature with few nodes on plane algebraic curves, on the plane and in higher dimensions, Journal of Complexity (45), pp. 22–54, 2018.